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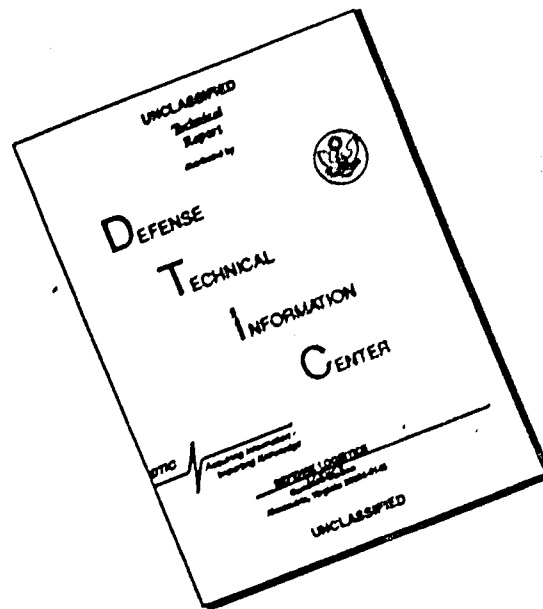
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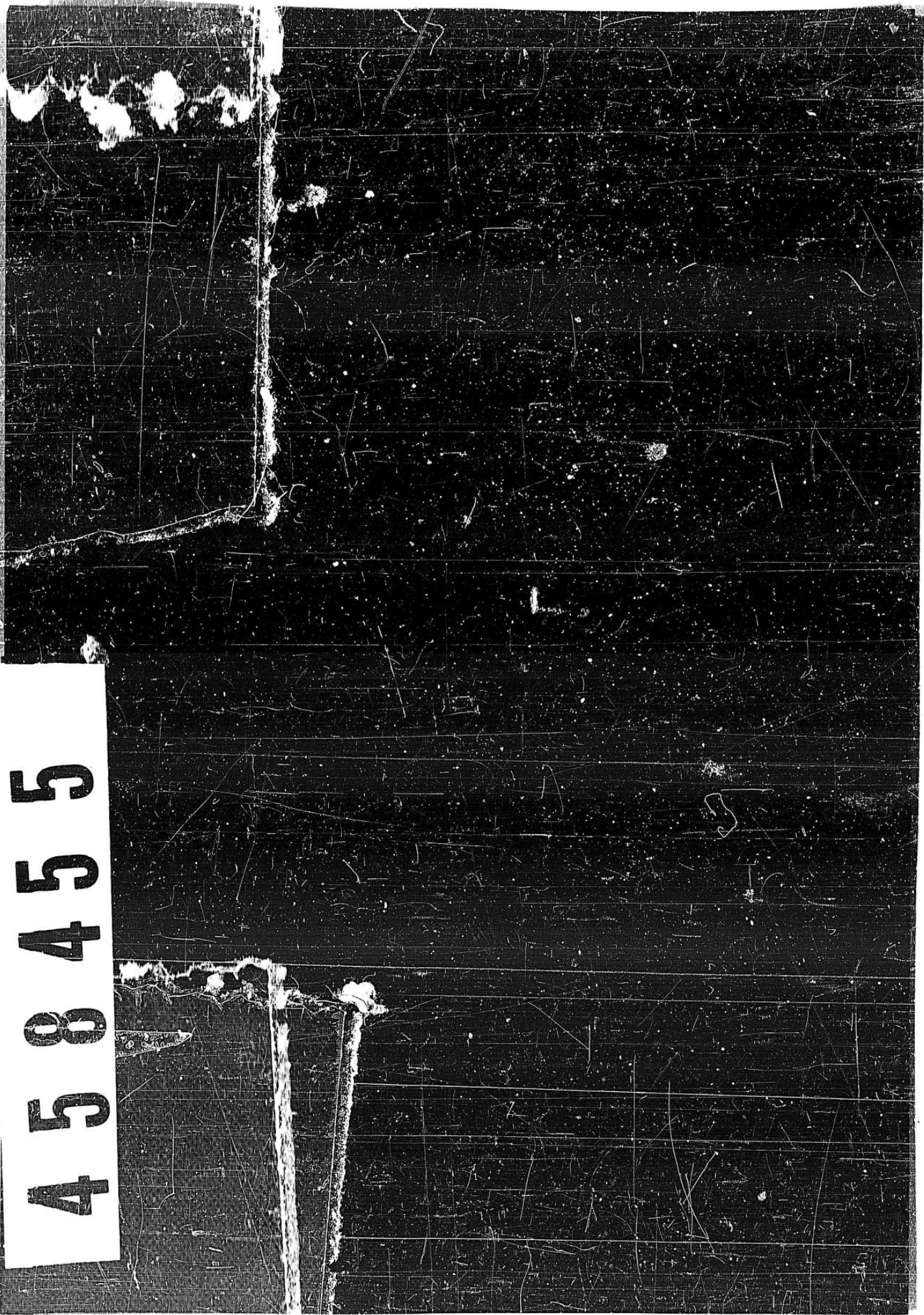
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ERRATA

8 June 65

**FINAL REPORTS OF THE  
WEAPON SYSTEM EFFECTIVENESS INDUSTRY ADVISORY COMMITTEE (WSEIAC)**

AFSC-TR-65-1

AFSC-TR-65-2 (Vols I, II & III)

AFSC-TR-65-3

AFSC-TR-65-4 (Vols I & III)

(Note: This Errata sheet applies to each of the above reports.)

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*Atch 2*

WEAPON SYSTEM EFFECTIVENESS  
INDUSTRY ADVISORY COMMITTEE (WSEIAC)

FINAL REPORT

of

TASK GROUP II

PREDICTION - MEASUREMENT  
(CONCEPTS, TASK ANALYSIS, PRINCIPLES OF MODEL CONSTRUCTION)

## FOREWORD

This is Volume II of the final report of Task Group II of the Weapon System Effectiveness Industry Advisory Committee (WSEIAC). It is submitted to the Commander, AFSC, in partial fulfillment of Task Group II objectives cited in the committee Charter. The final report is contained in three separate volumes:

Volume I contains an overview of Task Group II findings, including a summary of Volumes II and III, conclusions, and recommendations.

Volume II contains a discussion of effectiveness concepts, a description of specific tasks required to evaluate effectiveness, and a detailed example illustrating the method.

Volume III contains descriptions of effectiveness analysis methods applied to four typical Air Force systems using the techniques described in Volume II.

The membership of Task Group II was as follows:

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Mr. I. Doshay	Space General Corporation
Dr. B. J. Flehinger	IBM - Thomas J. Watson Research Laboratories
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Mr. H. J. Kennedy	ARINC Research Corporation
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Mr. M. M. Tall	Radio Corporation of America
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Other task group reports submitted in fulfillment of the committee's objectives are:

AFSC-TR-65-1	Final Report of Task Group I "Requirements Methodology"
AFSC-TR-65-3	Final Report of Task Group III "Data Collection and Management Reports"

AFSC-TR-65-4 Final Report of Task Group IV  
"Cost-Effectiveness Optimization"  
AFSC-TR-65-5 Final Report of Task Group V  
"Management Systems"  
AFSC-TR-65-6 Final Summary Report  
"Chairman's Final Report"

Publication of this report does not constitute Air Force Approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

APPROVED

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## WSEIAC CHARTER

In order that this report of Task Group II may be studied in context with the entire committee effort, the purpose and task group objectives as stated in the WSEIAC Charter are listed below:

### Purpose

The purpose of the Weapon System Effectiveness Industry Advisory Committee is to provide technical guidance and assistance to AFSC in the development of a technique to apprise management of current and predicted weapon system effectiveness at all phases of weapon system life.

### Task Group Objectives

Task Group I - Review present procedures being used to establish system effectiveness requirements and recommend a method for arriving at requirements that are mission responsive.

Task Group II - Review existing documents and recommend uniform methods and procedures to be applied in predicting and measuring systems effectiveness during all phases of a weapon system program.

Task Group III - Review format and engineering data content of existing system effectiveness reports and recommend uniform procedures for periodically reporting weapon system status to assist all levels of management in arriving at program decisions.

Task Group IV - Develop a basic set of instructions and procedures for conducting an analysis for system optimization considering effectiveness, time schedules, and funding.

Task Group V - Review current policies and procedures of other Air Force commands and develop a framework for standardizing management visibility procedures throughout all Air Force commands.

## ABSTRACT

Concepts of system effectiveness including the three principal terms, availability, dependability, and capability, are presented. Eight specific tasks required to evaluate effectiveness during any phase of system life are presented. A mathematical routine appropriate to effectiveness model construction is described. Using the above task analysis and the model framework, a hypothetical example is presented. Results of the evaluation illustrate effectiveness analysis methods and possible alternate decisions available. Application of simulation methods to the example are discussed. The appendixes contain summaries of four typical examples of the application of effectiveness evaluation methods to various Air Force systems (presented in detail in Volume III). An airborne avionics system, an intercontinental ballistic missile system, a long range radar surveillance system, and a spacecraft system are described.

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## SECTION I

### INTRODUCTION

The design and development of military systems has traditionally crowded the state of the art in materials, devices, and physical principles. In recent times, designers have been faced simultaneously with increasingly novel demands and ever more acutely limited test data. Performance requirements invariably include severe reaction and response time limits which cannot be met without a close integration of personnel, procedures, and hardware. At the same time program cost reductions, accelerated development schedules, and lack of opportunity for complete system tests prior to operational deployment have combined to reduce the opportunity to obtain extensive operational usage data, either in kind or quantity. Accordingly, what was once considered merely desirable is now mandatory -- an integrated methodology of system program management utilizing all available data both to pinpoint problem areas and to provide a numerical estimate of system effectiveness during all phases of the system life cycle.

It is the specific objective of Task Group II to "recommend methods and procedures for measuring and predicting system effectiveness during all phases of a program." Mathematical models utilizing analytical methods and machine simulation programs are an essential part of an integrated methodology.

Task Group II adopted a framework for system effectiveness evaluation based on three factors:

- (1) availability (readiness)
- (2) dependability (reliability)
- (3) capability (performance)

This framework was organized into a specific analytical structure, the use of which is illustrated in several examples.

The report discusses the general concepts associated with system

effectiveness. This discussion is followed by a description of the tasks which must be performed in order to arrive at a numerical estimate of system effectiveness, and to obtain insight into the controllable factors of the system that influence effectiveness. Finally, a tutorial example is given, illustrating methods of analysis in the formal analytical framework adopted by Task Group II.

## SECTION II

### GENERAL CONCEPTS

This section introduces concepts of system effectiveness evaluation. Specific definitions as they are employed in model construction are presented in Section IV.

System Effectiveness (Reference 1) is a measure of the extent to which a system may be expected to achieve a set of specific mission requirements and is a function of availability, dependability, and capability.

Availability is a measure of the system condition at the start of a mission and is a function of the relationships among hardware, personnel, and procedures.

Dependability is a measure of the system condition at one or more points during the mission; given the system condition(s) at the start of the mission and may be stated as the probability (or probabilities or other suitable mission oriented measure) that the system (1) will enter and/or occupy any one of its significant states during a specified mission and, (2) will perform the functions associated with those states.

Capability is a measure of the ability of a system to achieve the mission objectives; given the system condition(s) during the mission, and specifically accounts for the performance spectrum of a system.

The objectives of system effectiveness evaluation are to:

- (1) Evaluate system designs and compare alternative configurations.
- (2) Provide numerical estimates for use in defense planning.
- (3) Provide management visibility at every phase of a system's life cycle of the extent to which the system is expected to meet its operational requirements (SOR).
- (4) Provide timely indication of the necessity for corrective actions.
- (5) Compare the effect of alternative corrective actions.



SECTION III  
DESCRIPTION OF TASKS REQUIRED TO  
EVALUATE SYSTEM EFFECTIVENESS

3.1 Introduction

This section of the report describes eight tasks that must be performed in evaluating effectiveness. These tasks are discussed in terms of the requirements and available information during the four phases of system life: (1) conceptual; (2) definition; (3) acquisition; and (4) operational.

3.2 Phases of System Life

The objectives of each of the four phases of system life are described below.

3.2.1 Conceptual Phase

The objectives are to establish a feasible technical approach for satisfying a given requirement; to evaluate whether the approach is worth pursuing or whether the military requirement should be satisfied in another manner. The phase extends from determination of a broad objective or need to Air Force approval of the Program Change Proposal covering the Definition Phase.

3.2.2 Definition Phase

The objectives are to select and define the specific system configuration, to establish performance specifications, to provide cost and schedule estimates and to confirm the desirability of acquiring the system for use. This phase is initiated by System Definition Directive and ends with issuance of a System Program Directive.

3.2.3 Acquisition Phase

The objectives are to carry out detailed design and development, conduct category tests, and procure required quantities of hardware. The period starts after issuance of the System Program Directive and ends with acceptance by the user of the last operating unit in a certain series or until

the SOR has been demonstrated through Category I testing and all required updating changes resulting from the testing have been identified, approved, and placed on procurement, whichever occurs later.

#### 3.2.4 Operational Phase

The objective is to employ the procured system in an effective manner. This phase begins with acceptance by the user of the first operating unit and continues until final disposition of the system. It overlaps the Acquisition Phase.

Figure 1, extracted from AFR 375-1, "Management of System Programs," shows the four phases in terms of the Air Force decision process.

### 3.3 Tasks

Eight tasks used in evaluating effectiveness are listed below:

- (1) Mission definition
- (2) System description
- (3) Specification of figure(s) of merit
- (4) Identification of accountable factors
- (5) Model construction
- (6) Data acquisition
- (7) Parameter estimation
- (8) Model exercise.

The tasks are described below and are followed by a discussion of how they relate to the four phases of system life.

Figure 2 shows the eight tasks leading to an evaluation of effectiveness.

#### 3.3.1 Mission Definition

The mission definition is a precise statement of the intended purpose(s) of the system and of the environmental conditions (natural and man-made) under which it is required to operate.

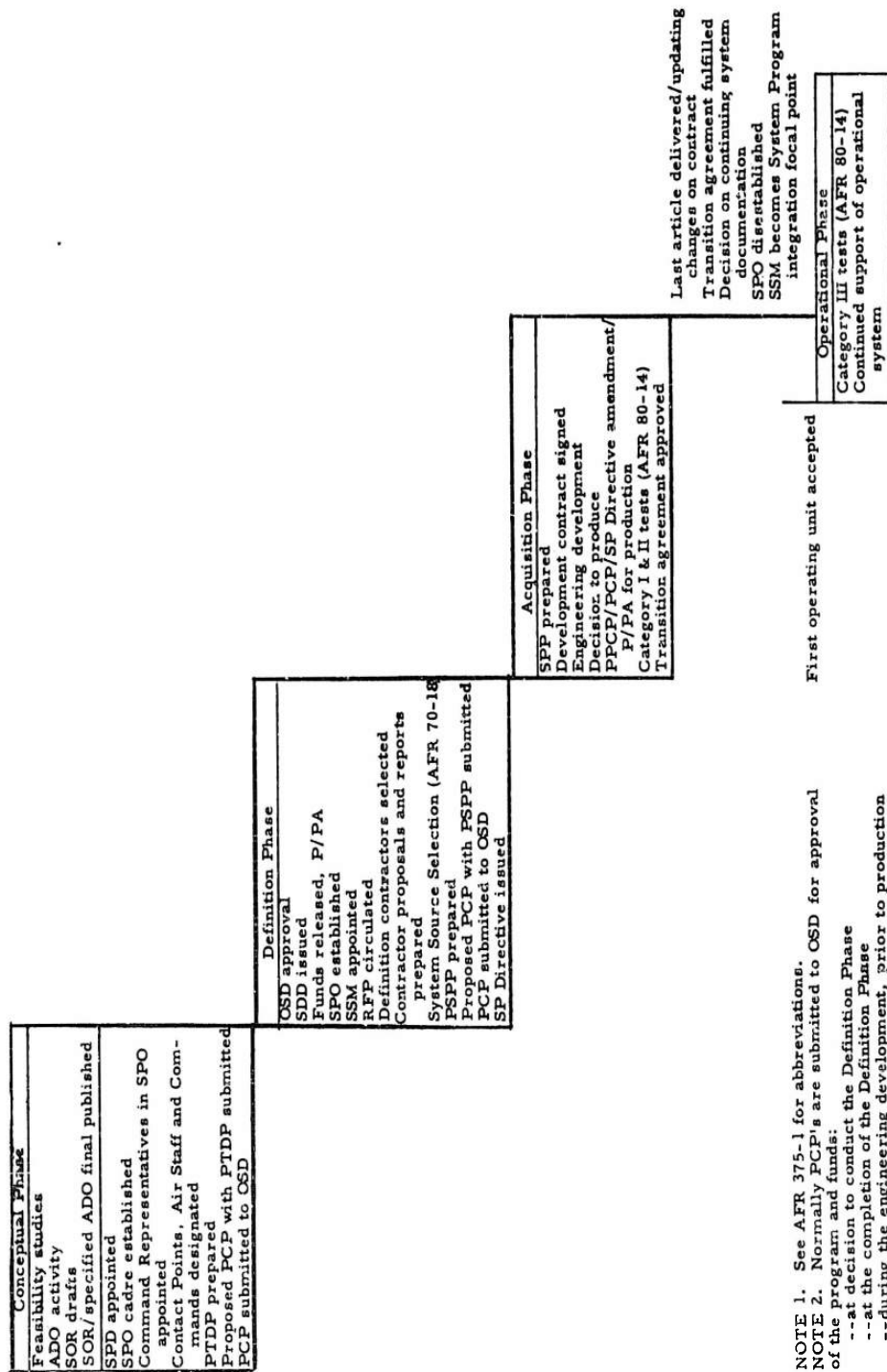


FIGURE 1. SYSTEM LIFE CYCLE

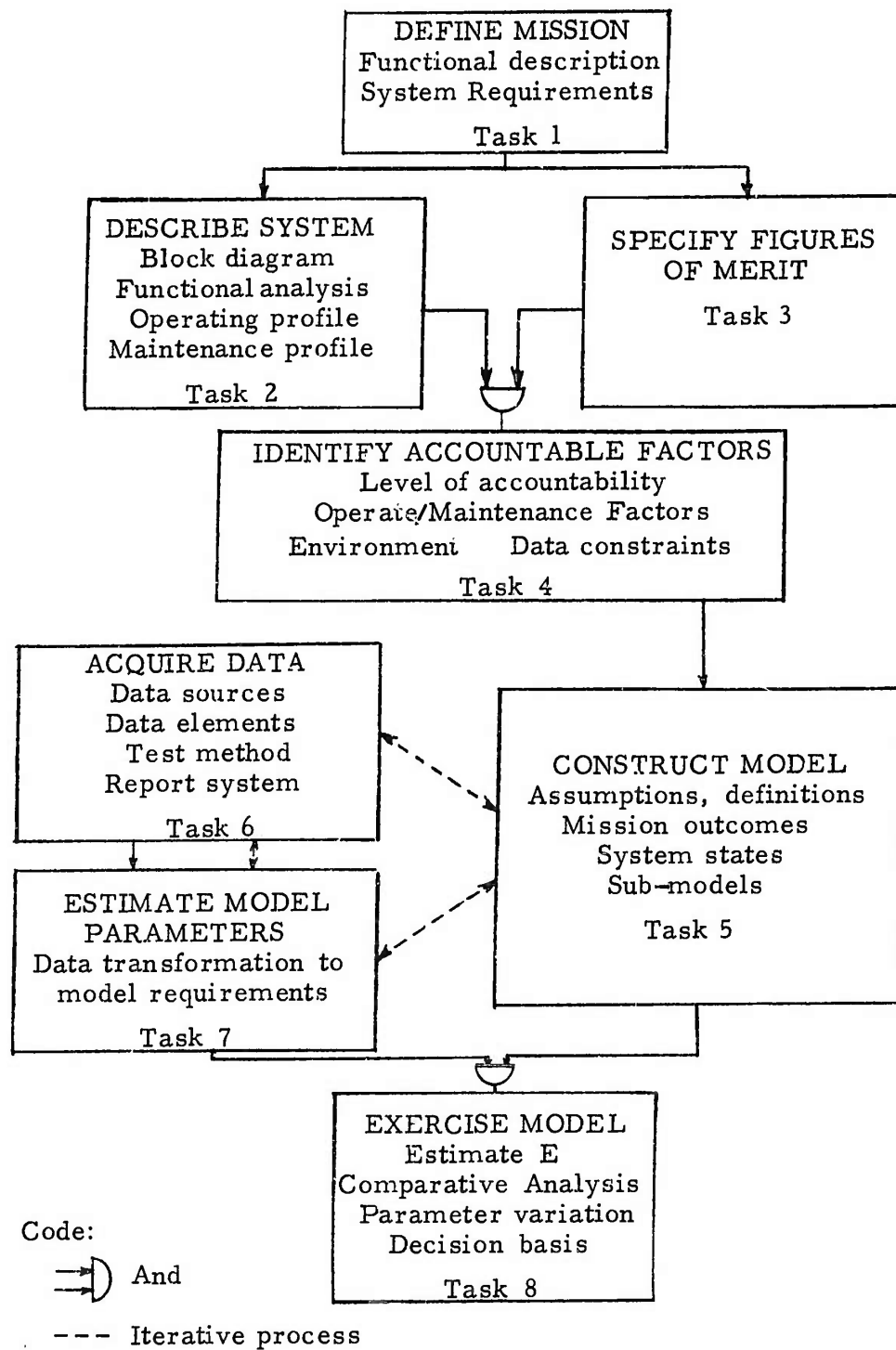


FIGURE 2. PRINCIPAL TASKS REQUIRED FOR EVALUATION OF SYSTEM EFFECTIVENESS

### 3.3.2 System Description

#### 3.3.2.1 General Configuration

The major hardware components of the system must be described and their functions defined.

#### 3.3.2.2 System Block Diagram

A block diagram of the system should be constructed showing signal flow and redundancy (Reference 2).

#### 3.3.2.3 Mission Profile

A time-line analysis showing the sequence of events from initiation of each mission to its completion should be prepared. This delineation may split the mission into a number of discrete time intervals during which different functions are being performed. The components used in each of these intervals must be identified, along with their contribution to mission success.

#### 3.3.2.4 Mission Outcomes

The principal events that might result from a mission must be selected and differentiated. In some cases, it is sufficient to differentiate between successful fulfillment of the purpose and failure to fulfill that purpose. In other cases, there are possibilities of partial success or even continuous gradations of outcomes ranging from total success to total failure.

### 3.3.3 Specification of Figure(s) of Merit

In general, a figure of merit is any index which indicates the quality of a system. In the simplest case it may be a measured physical quantity, such as range or payload. On the other hand, it may be a calculated quantity based on measurement, such as mean down time or mean time between maintenance actions. Lastly, it may be a predicted quantity based on measurement and/or simulation. For example, "the probability that a system can meet an operational demand at a random point in time while under attack," will require prediction since there will be some

uncertainty about the attack environment.

Figure(s) of Merit serve to indicate what can be expected from the system. They must be in an operationally-oriented form that can be readily understood and utilized in planning (References 3 and 4). Where the number of significantly different mission outcomes is small, the probabilities of each of these outcomes can be the appropriate figures of merit. When the number of mission outcomes is large or when a continuous range of outcomes requires consideration, a measure of relative "adequacy" may be assigned to each possible outcome, and the expected "adequacy" should be used as a figure of merit.

System Effectiveness is defined as the vector of the specified figure(s) of merit.

#### 3.3.4 Identification of Accountable Factors

Accountable factors are those specific factors which are known or suspected to have a significant influence on the figure(s) of merit. All assumptions which are made in regard to these factors must be explicitly stated. Thus, it is essential to preface any analysis by a list of the assumptions made concerning the intrinsic failure and repair characteristics of the components (e.g., exponential distributions), the maintenance policies in effect (e.g., preventive maintenance schedules, checkout procedures), and the environmental conditions under which the system is to operate (e.g., temperature extremes, vibration, enemy countermeasures, etc.). Since the relative importance of a specific factor is a strong function of the phase of system evolution, a periodic review of this listing should be made to ascertain that the model contains at least those factors which can be influenced by the decision maker at that particular point in the system life.

Table I is a checklist for identification of accountable factors.

#### 3.3.5 Model Construction

The model is a technique for combining the information developed in the prior four tasks in order to estimate system effectiveness. The model serves as a probabilistic representation of the events which may

occur prior to and during a mission. It relates these possible events to the levels of performance adequacy which may be expected for the mission.

TABLE I  
CHECKLIST FOR IDENTIFICATION OF ACCOUNTABLE FACTORS

System Hardware Description	Spares
. Modes of operation	. Provisioning
. Hardware organization	. Storage
Compatibility	. Packaging
(e.g., Electromagnetic Compatibility)	Transportation
Survivability	Support Equipment
Vulnerability	. Test
Deployment	. Transport
Geographic Factors	. Maintenance
. Deployment	. Facilities
. Geology	Procedures/Policies
. Climate	. Operating
. Atmospheric phenomena	. Repair
Personnel	. Inspection/Maintenance
. Operating	. Testing
. Maintenance	System Interfaces
	. Support systems
	. Force mix
	. Strategic Integrated Operations Plan (SIOP)

The first step in model construction is to describe the significantly different system "states" in which the mission may be carried out. System "states" are distinguishable conditions of the system which result from events occurring prior to and during the mission. For example, the condition in which all system hardware is functioning within design specifications is one state. The condition in which the system is completely inoperable due to hardware, personnel, or procedures is a state at the other extreme. The conditions of partial system operation due to defects of hardware, personnel, or procedures are represented by the intermediate system states. It should be evident that the system can make transitions from state to state during a mission. The time-line analysis performed in accordance with Section 3.3.2.3 above, may have split the mission into a number of discrete time intervals during which different functions are being performed and different portions of the system's hardware are being used. For each discrete time interval, a set of significant states appropriate to the function being performed during that interval may be defined.

The next step is to relate to accountable factors the probabilities of each of the sets of significant states which are appropriate at the beginning of the mission. This array of probabilities is called the availability vector. For each succeeding time interval, an array of state probabilities is related to accountable factors. These probabilities are dependent or conditional on the effective state during the previous time interval. For example, where no repair is possible, a failure in one interval predetermines the possible states in the succeeding intervals. These arrays of conditional probabilities are called the dependability matrices.

A simplified method of analysis which is generally employed, defines the significantly different system states over the entire mission rather than for each discrete time interval. The array of state probabilities at the beginning of the mission still yields the availability vector. However, the dependability matrix contains the probabilities of the effective states throughout the mission conditional on the initial states.



For the simplified method, the next step is the construction of the capability matrix. This is an array of numbers which are a measure of the ability of a system to achieve the mission objectives; given the system condition(s) during the mission. This array of numbers (vector or matrix) specifically accounts for the performance spectrum of the system. This occurs, for example, when the accumulation of subsystem performance deviations, each within acceptable tolerances, results in a bomb drop being wide of the mark. In this case, there has been no specific subsystem malfunction, but a system malfunction (or performance degradation) due to the unlikely combination of within tolerance variations of the subsystem. There may, therefore, be a continuous spectrum of possible mission results, none of which is an unequivocal failure or success. The capability matrix represents the expected figures of merit for the system. Each element of the matrix is an expected figure of merit conditional on carrying out the mission in a given effective state. The matrix has a column for each figure of merit selected in accordance with Section 3.3.3 and a row for each effective state.

Defining the capability matrices for each of the time intervals discussed in the earlier general case becomes quite complex. It is for this reason, that the simplified approach is used so generally. A discussion on defining and combining the capability matrices for the general case will be found in A Model Framework for System Effectiveness.<sup>1/</sup>

Model construction has been described in four steps: (1) state description; (2) determination of availability vector; (3) determination of dependability matrix; and, (4) determination of capability matrix. In specific system cases, it may be impractical to construct the model exactly following these steps. For example, it may not be desirable or practical to separate the dependability and capability matrices in some instances. However, the four steps do serve as a useful guide in constructing the model.

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<sup>1/</sup> See Volume III (Technical Supplement) A Model Framework for System Effectiveness, H. S. Balaban, ARINC Research Corp., prepared for WSEIAC, Task Group IV.

The available informational inputs and the decisions to be made on the basis of the outputs strongly influence the structure of the model. These models may also be subdivided by level of system evaluation such as the overall system, subsystems, equipments and module (or piece part) levels. The level of evaluation depends on the objective of the particular evaluation and the available informational inputs. The steps of model construction are described in mathematical detail in Section IV.

### 3.3.6 Data Acquisition

The accountable factors determined in Section 3.3.4 and the model detail level determined in Section 3.3.5 imply data element requirements. These must be specifically identified in the data acquisition task. The specification of data elements is a two-way proposition. The analyst can answer only those questions for which there is an "adequate" data base. "Adequate" to the analyst may not be consistent with constraints of time, cost or schedules imposed upon the project manager.

The source of data elements and the method of collection (i. e., from standard Air Force reporting forms, from category tests performed by a contractor, etc.) should be stated. Data may be obtained from published reference material such as the Interservice Data Exchange Program (IDEP), Mil Hdbk 217, or other generic data sources<sup>2/</sup> including historic information from earlier systems. A listing of typical data elements required is shown in Table II<sup>3/</sup>. Typical data elements are time to failure and repair time of components (Reference 5). Care should be taken to ensure that all parameters used in models have an available data source.

The completeness, the appropriateness, and the compatibility of available sources of data constitute the largest cause for difference in the evaluation of effectiveness from one phase to another in the system life.

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<sup>2/</sup>See BIBLIOGRAPHY

<sup>3/</sup>Op. Cit., Task Group III Final Report

TABLE II  
TYPICAL DATA ELEMENT REQUIREMENTS

<p>(1) General Identification information (name, enclosure, etc.)</p> <p>(2) Time information (chronological time and sequence of events).</p> <p>(a) Operating Times</p> <ol style="list-style-type: none"> <li>1) mission time and phases</li> <li>2) non-mission time               <ol style="list-style-type: none"> <li>a) checkout and test time</li> <li>b) full on standby</li> <li>c) partial on standby</li> </ol> </li> </ol> <p>(b) Non-operating Times</p> <ol style="list-style-type: none"> <li>1) off, no demand               <ol style="list-style-type: none"> <li>a) storage</li> <li>b) free time</li> </ol> </li> <li>2) downtime (when in demand)               <ol style="list-style-type: none"> <li>a) repair time</li> <li>b) logistic time (spares, transportation, queuing, other support-oriented items)</li> <li>c) administrative time (training, other cause of personnel non-availability)</li> <li>d) effect of emergency procedures</li> </ol> </li> </ol> <p>(3) Event information</p> <p>(a) Failure events</p> <ol style="list-style-type: none"> <li>1) identification of failure</li> <li>2) effect on mission capability               <ol style="list-style-type: none"> <li>a) critical</li> <li>b) non-critical</li> </ol> </li> <li>3) repairable during mission</li> <li>4) how detected</li> <li>5) failure classification               <ol style="list-style-type: none"> <li>a) primary</li> <li>b) secondary</li> </ol> </li> <li>6) cause classification               <ol style="list-style-type: none"> <li>a) design</li> <li>b) operational environment                   <ol style="list-style-type: none"> <li>1) controlled</li> <li>2) uncontrolled</li> </ol> </li> <li>c) personnel induced                   <ol style="list-style-type: none"> <li>1) supplier</li> <li>2) user</li> </ol> </li> <li>d) time-dependent</li> </ol> </li> </ol>	<p>(b) Maintenance events</p> <ol style="list-style-type: none"> <li>1) classes of maintenance (includes monitoring and system exercising)               <ol style="list-style-type: none"> <li>a) corrective maintenance                   <ol style="list-style-type: none"> <li>1) scheduled</li> <li>2) unscheduled</li> </ol> </li> <li>b) preventive maintenance                   <ol style="list-style-type: none"> <li>1) scheduled</li> <li>2) unscheduled</li> </ol> </li> </ol> </li> <li>2) event information               <ol style="list-style-type: none"> <li>a) type of action                   <ol style="list-style-type: none"> <li>1) replacement</li> <li>2) adjustment</li> <li>3) repairs                       <ol style="list-style-type: none"> <li>a) in place</li> <li>b) other location</li> </ol> </li> </ol> </li> <li>b) manhours expended (minimum number of personnel required)</li> <li>c) level of personnel</li> <li>d) adequacy of equipment and tools</li> <li>e) availability and quality of spares</li> <li>f) adequacy of facilities</li> <li>g) adequacy of technical data</li> <li>h) adequacy of maintenance action</li> </ol> </li> </ol> <p>(4) Capability information</p> <ol style="list-style-type: none"> <li>(a) Determine that the system complies with specified requirements</li> <li>(b) Determine these significant performance parameters which contribute to mission success</li> <li>(c) Based on performance of actual mission and/or inferred performance from system simulations, measure the capability of the system to accomplish mission objectives within a stated environment, and when the system performs in accordance with specifications. This is a difficult measurement to obtain on many systems. Therefore, calculations of inferred or expected performance will be required from system simulations.</li> <li>(d) To the degree possible, measure or calculate the effect of various environments on the system capability.</li> </ol>
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During the conceptual phase, heavy reliance must be placed on sources of generic data, on results learned from similar systems and on application of basic knowledge about physical laws appropriate to the system concept. Late in the development period and during operational use, a great deal more data appropriate to the evaluation can be made available. However, testing is expensive, and present data retrieval systems are not well suited to effectiveness analysis. These factors strongly indicate that planning for effectiveness evaluation must be an integral and carefully identified portion of all phases of the system life cycle.

#### 3.3.7 Parameter Estimation (Reference 6)

Processing the data elements to derive numerical estimates for the parameters of the model is the next task. The analytical techniques used to reduce the data are referred to here as "parameter estimation" techniques.

The specific methods used depend upon:

- (1) The nature of the quantity being estimated
- (2) The control which can be exerted over the physical mechanisms which generate the data
- (3) The format of data collection.

The simple case that occurs when a control population is tested at one environmental stress level for the exponential or Weibull distributions of failures has been extensively treated in the literature. The problems that arise when the data are fortuitously collected at a variety of environmental stress levels (as is the usual case with field-generated data) have been much less thoroughly investigated. In particular, the question of accuracy of estimation versus sample size as a function of the method of estimation requires further investigation.

#### 3.3.8 Model Exercise

The system effectiveness vector is now calculated using the model equations. Variations in accountable factors may be made to determine the effect on the end result. System change analysis may also

be accomplished for product improvement or optimization, and the influence on system effectiveness may be estimated. Completion of the model and its exercise forms the principal vehicle for effectiveness decision-making. These decisions may vary from electing to proceed from the Conceptual to the Program Definition Phase of a project, or deciding to modernize an existing system by replacement of a major component.

Section IV of this volume presents a detailed treatment of effectiveness model development. An illustrative example is described which follows each of the eight tasks.

### 3.4 Discussion of Tasks by Program Phase

The objectives of effectiveness evaluation will differ depending on the program phase. In the Conceptual Phase an estimation of probable effectiveness may be one of the most important factors influencing the decision whether or not to proceed to program definition. Low levels of expected effectiveness may indicate desirability of considering alternate approaches to meeting the requirement. In the Program Definition Phase, effectiveness considerations should influence the choice of major system elements and their configuration. Whether redundancy will be required, how the major elements of hardware, personnel, procedures, and logistics may be optimized for maximum effectiveness, or how to meet minimum acceptable levels at least total cost are questions that require resolution.

In the Acquisition Phase, updated estimates of effectiveness should be made frequently to measure achievement of, or growth toward, the specified figure of merit. Detailed tradeoffs within the subsystems, equipments, and modules (parts) will be required to meet the requirements or to determine a best compromise (resource allocation).

Effectiveness evaluation during the Operational Phase may form the basis for design changes, a product improvement program, or modification in the support/logistics structure. Comparisons with other existing or proposed systems may be made, and decisions as to force structure, deployment, etc., may be required to optimize mission objectives. Regardless of the objective of the evaluation, a logical procedure should be followed during

the process. Certainly the amount of detail will differ greatly depending on the objective and available information, but the formalized steps should nevertheless be followed. In actual practice information obtained during one phase may remain constant or may be augmented and refined during subsequent phases.

The following paragraphs provide some additional discussion of the eight tasks listed above as they may differ during the four major system phases.

#### 3.4.1 Mission Definition

Generally the mission definition originates from a need or requirement that forms the basis for entering the Conceptual Phase. By the end of the Conceptual Phase the Specific Operational Requirements form the basis for mission definition. This definition may be clarified during the Program Definition Phase and more detail relative to alternatives may become known. The defined mission changes little during the subsequent phases except as external factors or constraints arise, such as a major state-of-the-art breakthrough, allowing an extension of the mission objectives, or a new enemy capability that may require modifications to the original mission.

#### 3.4.2 System Description

During the Conceptual Phase very little detail will normally be available describing hardware and software elements of the system. Information will probably include a block diagram identifying major system elements, such as tracking radar, receiver, display unit, etc.. During program definition, the major hardware end items will be defined, modes of operation determined and a time line of the mission profile developed. Physical and environmental factors can be estimated and some knowledge of the maintenance support and logistics plan will be available. During the Acquisition Phase, complete detail of the hardware system down to piece part identification will become available. The detailed system operating plan and maintenance task analysis will have been developed. By the early Operational Phase, complete descriptive information on all hardware,

procedures, operation and maintenance schedules, and logistics plans should be available. Effectiveness evaluation will require continued updating of this information as continued changes are proposed and adopted. Frequently, each new system placed in operation will have a unique configuration.

For an illustration of the successive detail required in system description, see Example B, "Intercontinental Ballistic Missile Squadron," in Volume III.

#### 3. 4. 3 Specification of Figure(s) of Merit (FOM)

The selection of a figure of merit will largely depend upon the particular system under evaluation. For a radar this may be the probability of target detection or the probability of successful (accurate) track. For an ICBM fleet this figure may be the expected number of targets destroyed. Other examples of appropriate figures of merit are described in Section V, "An Illustrative Example," and in the Technical Supplement, Volume III.

At the Conceptual Phase, the FOM chosen will be general. It may consist of a qualitative statement as to the probability of accomplishing the system requirement or mission.

During the Program Definition Phase, the FOM's and their prime factors and subfactors will be identified with more complete statements relative to the units of measurement and appropriate conditions under which the evaluation is made. As the system description becomes more detailed during the Acquisition Phase, FOM's and their prime A, D, C (availability, dependability, and capability) factors must be defined at various levels ranging from system to subsystem to equipment on down to the module and piece part level.

During the Operation Phase little change will normally be made to the finally selected and refined figures of merit.

#### 3. 4. 4 Identification of Accountable Factors

This task involves clear recognition of the data constraints that

may be imposed due to the phase of the program, the cost of acquiring data, and the level of detail that is required in the effectiveness evaluation. Sources of required data must also be identified. For example, will the analysis require gross system information, or will detail down to the piece part level be needed? Where will this data come from?

In the Conceptual Phase, very little information on personnel, procedures, hardware, and logistics will normally be available. Again, heavy dependence will be placed on estimates and extrapolations from other related programs. The accountability will generally be to the system functional block diagram level. Crude estimates of personnel requirements, the maintenance concept, logistic considerations, etc., can be made.

During the Program Definition Phase, accountability will normally extend to the principal end-items. Information needed will include:

- (1) the environmental conditions surrounding both the use of, and data collection on, these items. (Reference 7)
- (2) generic reliability and maintainability data.
- (3) failure rate estimates on parts for which generic data does not exist.
- (4) detailed support policy.
- (5) numbers and skill levels of personnel.
- (6) maintenance facilities.
- (7) maintenance equipment and tools.
- (8) critical logistic support considerations.
- (9) system monitoring and exercising requirements.

During the Acquisition Phase, additional factors and refinement of previous information should be obtained, such as:

- (1) generic failure data for specific parts in specific applications.
- (2) failure experience during developmental testing.
- (3) operating time during design and development.
- (4) modifications to the mission.
- (5) refined support information.



(6) clarification of operational environment.

In the Operational Phase, it is generally possible to obtain complete information on all accountable factors. However, the method of obtaining the information may be costly, may interfere with system operation, or may not be relevant to the available decisions and degrees of freedom of action. Seldom at the Operational Phase will a complete detailed analysis of effectiveness be made down to the piece part level, except where a major contribution to final system effectiveness is isolated or suspected.

#### 3. 4. 5 Model Construction

Section IV of this volume traces the steps of model construction in detail.

The level of the detail in model construction is directly related to the system phase. During the Conceptual Phase, the model may be simple.

During the Program Definition Phase, the detail should extend to the principal system hardware elements with submodels at the subsystem level and for each of the principal effectiveness parameters.

During development and early production (Acquisition Phase), the model will normally contain the most detail, since alternate decisions and tradeoffs, design changes, and continuing configuration updating will involve great detail in the relevant information from which the model is constructed.

During the Operational Phase, model inputs may be limited to the information from Air Force reporting systems, such as the AFM 66-1 Maintenance Data Collection System, AFM 65-110 Aircraft Status Reporting, and like data sources.

#### 3. 4. 6 Data Acquisition and Parameter Estimation

For this task, the largest difference in detail among the four system phases is apparent. Data requirements, test and observation methods to be employed, and data collection and processing systems are obviously tailored to the particular situation and the inevitable constraints. In the Conceptual Phase, this information may come entirely from previous

systems, be derived from generic sources, or developed from basic physical laws.

During the Program Definition Phase, parameters are generally predicted using generic data sources, or historic records of similar hardware items.

During development and production, this information is refined and supplemented by proofing and category tests at the system and equipment level and by more extensive in-process tests of assemblies and parts by their respective manufacturers. Assumptions or other provisions must be stated that relate these parameter estimates to the probable end-use environment.

During the Operational Phase, parameter estimation will be made from actual operating, failure, and maintenance data. However, the usefulness of present information for this purpose is questionable. Carefully directed observation programs are needed that are tied closely to the model requirements so that the data derived may be effectively translated into the required parameters.

#### 3.4.7 Model Exercise

The extent of model exercise and the amount of information derived is of course dependent upon the previous tasks. Results of the model exercise by phases may vary from a single point estimation of effectiveness with low confidence (Conceptual Phase) to an elaborate ream of information on systems, subsystem, equipment, etc., including estimates of effectiveness factors and their elements as well as parameter variation analysis and system change analysis (during the later portion of the Acquisition Phase).

## SECTION IV

### MATHEMATICAL MODEL CONSTRUCTION

#### 4.1 Mathematical Expression for Effectiveness

In Section 3.3.3, System Effectiveness  $E$  was defined as the vector of figures of merit specified for a given system. In Section 3.3.5, a structure for evaluating this vector was developed. This structure is based on an enumeration of a number  $N$  of significantly different system states,  $1, \dots, n$ . The structure is composed of three distinct parts, an availability vector  $\bar{A}$ , a dependability matrix  $[D]$ , and a capability matrix  $[C]$ . The elements of  $E$ ,  $A$ ,  $D$ , and  $C$  are defined respectively as follows:

$e_k$  is the value of the  $k^{\text{th}}$  figure of merit

$a_i$  is the probability that the system is in state  $i$  at the beginning of the mission

$d_{ij}$  is the probability that the effective state of the system during the mission is  $j$ , given that the mission was begun in state  $i$

$c_{jk}$  is the value of the  $k^{\text{th}}$  figure of merit, conditional on effective system state  $j$ .

Based on these definitions, we may write

$$E = \bar{A}' [D] [C]$$

where  $\bar{A}'$  is the transpose of  $\bar{A}$ ,

or

$$e_k = \sum_{i=1}^n \sum_{j=1}^n a_i d_{ij} c_{jk}.$$

#### 4.2 Availability Vector

The availability vector  $\bar{A}'$  is a row vector  $[a_1, \dots, a_n]$  containing the probabilities of the various system states at that point in time when a mission begins.

We may illustrate the principles of availability analysis by considering a system for which the following assumptions are a satisfactory representation of reality:

- (1) The system is a single "black box" which fails at random times following repair;
- (2) When the system fails, it is immediately evident and repair is initiated at once. Repair completely renews the system;
- (3) The system is in operation when it is not in repair; and,
- (4) The demand for system use occurs sufficiently long after initial system installation so that it has undergone several failure-repair cycles.

For this system there are two possible states: (1) "operative" or (2) "failed" (in repair).

The availability vector  $\bar{A}'$  then consists of two components,  $a_1$  and  $a_2$ :

$$\bar{A}' = [a_1 \ a_2]$$

where

$a_1$  = probability that system is operable at a random point in time

$a_2$  = probability that system is in repair at a random point in time.

The probability that it is operating (i. e. , in State 1) will be given by

$$\begin{aligned} \text{Availability} &= a_1 \\ &= \frac{(\text{mean time to failure})}{(\text{mean time to failure}) + (\text{mean time to repair})} \end{aligned} \quad (\text{Eq. 1})$$

and the probability that it is in repair (i. e. , in State 2) will be given by

$$\begin{aligned} \text{Unavailability} &= a_2 \\ &= \frac{(\text{mean time to repair})}{(\text{mean time to failure}) + (\text{mean time to repair})} \end{aligned} \quad (\text{Eq. 2})$$

That is, our best guess for  $a_1$  is the expected fraction of time that the system will be non-failed and our best guess for  $a_2$  is the expected fraction of the time that the system will be in repair. Therefore, if we have the chronological past history of the system we can estimate  $a_1$  and  $a_2$  by

$$\hat{a}_1 = \frac{(\text{total time operating})}{(\text{total time operating}) + (\text{total time in repair})}$$

$$\hat{a}_2 = \frac{(\text{total time in repair})}{(\text{total time operating}) + (\text{total time in repair})}$$

However, not all systems are observed continuously. For example, consider a single unit that is checked out by a series of tests and then placed in an active, but unattended and unmonitored state for a constant period of time  $T$ . Assume that this cycle of checkout and standby repeats itself indefinitely. It may fail during the time  $T$ , and if it does, the failure will not be discovered until the time  $T$  has elapsed and a new series of checkouts is undertaken. We say that such a system is periodically checked, and we may express the availability of this system by,

$$\begin{aligned} a_1 &= \text{Availability} \\ &= \frac{(\text{expected time non-failed in } T)}{(\text{duration of } T) + (\text{time down in checkout/repair})} . \end{aligned}$$

This expression assumes that repair, if it occurs, extends the time down beyond that required to check out a non-failed system and that the expected time non-failed in  $T$  is not dependent upon the age of the system, although system aging (wearout) can be included by making a somewhat more complicated calculation.

Practical systems are usually composed of a number of single units or black boxes. Consider, for example, a system for which the following assumptions constitute a satisfactory approximation to reality during pre-mission life:

- (1) There are  $N$  units which fail independently with mean times to failure  $\bar{t}_{f_1}, \dots, \bar{t}_{f_N}$  that do not change with system age.

- (2) When any one of these units fails, it is immediately evident and repair is initiated at once (continuously monitored system with unlimited repair facilities). Repair times have stationary distributions with means  $\bar{t}_{r_1}, \dots, \bar{t}_{r_N}$ .
- (3) Any units which are not undergoing repair are in use.
- (4) The mission is started at a random point in time sufficiently long after initial system operation so that each unit has undergone several failure-repair cycles.

For this system, each unit has two possible states, operative (O) or undergoing repair (I). There are therefore, at most  $2^N$  system states, each of which may be represented by  $N$  binary digits corresponding to the component states, although a judicious lumping together of similar system states will reduce the total number of states which must be explicitly represented in the analysis.

The availability of any one unit of the system is given by this modification of Equation 1:

(Eq. 3)

$$\alpha_i = \frac{(\text{mean time to failure of } i^{\text{th}} \text{ unit})}{(\text{mean time to failure of } i^{\text{th}} \text{ unit}) + (\text{mean time to repair } i^{\text{th}} \text{ unit})}$$

We might define  $a_1$  as the probability that all  $N$  units are available. Thus,  $a_1$  would be calculated from Equation 3 by:

$$a_1 = \prod_{i=1}^N \alpha_i$$

The second state of the system might be defined as the probability that all units but the first are available. This would be calculated from Equation 3 by:

$$a_2 = (1 - \alpha_1) \prod_{i=2}^N \alpha_i$$

This process of combining the  $\alpha_i$  would be continued until all significant

system states had been accounted for.

Models for computing the elements of the availability vector must take into account failure and repair time distributions, preventive maintenance and miscellaneous down-time schedules, checkout procedures, personnel deployment, spare parts, supply facilities as well as transportation and various administrative actions. (References 8, 9, 10)

The following list of possible system states should be referred to when establishing availability models:

- (1) Assigned to alert/standby and non-failed
- (2) Assigned to alert/standby and failed in a manner detectable by field test
- (3) Assigned to alert/standby and failed in a manner undetectable by field test
- (4) On alert/standby and waiting for checkout/diagnosis
- (5) Off alert/standby and in checkout/diagnosis
- (6) Off alert/standby and waiting for spares.

#### 4.3 Dependability Matrix

The availability vector presents a picture of the condition in which the system is likely to be found at the beginning of a mission. The next step in analysis requires a representation of the system during the course of a mission, conditional on its state of readiness at the beginning.

The dependability matrix is a square array of numbers. In general, if there are  $n$  significant system states, the dependability matrix has  $n$  rows and  $n$  columns

$$[D] = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \dots & d_{nn} \end{bmatrix}$$

where

$$\sum_{j=1}^n d_{ij} = 1, \quad i = 1, \dots, n.$$

If no repair is possible during the mission some of the  $d$ 's will be zero. If the states are numbered in order of increasing degradation, the matrix is triangular

$$[D] = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ 0 & d_{22} & \dots & d_{2n} \\ \vdots & & & \\ 0 & 0 & \dots & d_{nn} \end{bmatrix}$$

The specific formulation of the dependability matrix will depend upon the effect of failures during the mission and whether or not repair is possible during the mission. For example, consider three different single unit systems which carry out missions of duration  $T$ , each having the following characteristics:

System a (for example, an ICBM)

- (1) No repair possible during mission
- (2) System state at end of mission is only important mission criterion

System b (for example, a G. C. A. system)

- (1) Repair is possible during mission
- (2) System state at end of return of air-craft from mission is only important mission criterion

System c (for example, doppler navigation radar)

- (1) Limited repair is possible during mission
- (2) Fraction of time out of commission during mission is mission criterion (e. g., positional accuracy degradation).



In these examples assume that the system must be in either one of two states at the time of demand (mission initiation): namely, operable (1) or failed (2). Thus, we consider a 2 x 2 dependability matrix:

$$[D] = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

We place the following interpretations on the  $d$ 's for systems a and b:

$d_{11}$  = The probability that the system is operable at the end of the mission, given that it was operable at the start of the mission

$d_{12}$  = The probability that the system is failed at the end of the mission, given that it was operable at the start of the mission

$d_{21}$  = The probability that the system is operable at the end of the mission, given that it was failed at the start of the mission

$d_{22}$  = The probability that the system is failed at the end of the mission, given that it was failed at the start of the mission.

Now, since there can be no repair for system a during the mission,  $d_{11}$  is simply the conventional reliability of the system, so that if failures are random and mean time to failure does not depend upon system age, the following relationship holds

$$d_{11}^{(a)} = e^{-\lambda T}$$

$\lambda$  = system failure rate

$T$  = duration of mission

and

$$d_{12}^{(a)} = 1 - d_{11} = 1 - e^{-\lambda T}$$

Also, since repair is not possible during the mission, a system failed at the initiation of the mission must still be failed at the end of the

mission; therefore:

$$d_{22}^{(a)} = 1$$

and

$$d_{21}^{(a)} = 0$$

Hence, the dependability matrix for system a is

$$[D]^{(a)} = \begin{bmatrix} e^{-\lambda T} & 1 - e^{-\lambda T} \\ 0 & 1 \end{bmatrix}$$

Although we interpret the d's in the same manner for system a and b, their calculation is not the same, since we allow for the possibility of repair during the mission for system b.

For simplicity, let us assume that times to failure after repair actions, and times to repair after failures are exponentially distributed for system b. This means that the probability of a failure or a repair in a small increment of time  $\Delta t$  can be expressed as

$$\text{probability of failure in time } \Delta t = \lambda \Delta t$$

$$\text{probability of a repair in time } \Delta t = \mu \Delta t$$

where

$$\lambda = \text{system failure rate}$$

$$\mu = \text{system repair rate.}$$

Then we may write:

$$P_1[t + \Delta t] = P_1[t] (1 - \lambda \Delta t) + P_2[t] \mu \Delta t.$$

This statement reads: [Probability that the system is operable at time  $t + \Delta t$ ], = [Probability that the system was operable at time  $t$ ], x [Probability that the system does not fail in the time increment  $\Delta t$ ] + [Probability that the system was failed at time  $t$ ] x [Probability that a repair is completed

in the time increment  $\Delta t$ . Subtracting  $P_1[t]$  from both sides of this equation, dividing both sides by  $\Delta t$ , and taking the limit as  $\Delta t \rightarrow 0$ , we have from the fundamental definition of a derivative, that

$$\begin{aligned} \lim_{\Delta t \rightarrow 0} \frac{P_1[t + \Delta t] - P_1[t]}{\Delta t} &= \frac{d P_1[t]}{dt} \\ &= -\lambda P_1[t] + \mu P_2[t] \end{aligned} \quad (\text{Eq. 4})$$

But since the system can only be either failed or non-failed,

$$P_1[t] + P_2[t] = 1 \quad (\text{Eq. 5})$$

Combining Equations 4 and 5

$$\frac{d P_1[t]}{dt} = -(\lambda + \mu) P_1[t] + \mu$$

The general solution to this differential equation is found to be

$$P_1[t] = \frac{\mu}{\lambda + \mu} \left[ 1 - e^{-(\lambda + \mu)t} \right] + P_1[0] e^{-(\lambda + \mu)t} \quad (\text{Eq. 6})$$

where  $P_1[0]$  is the probability that the system is operable at time  $t = 0$ . If we identify  $t = 0$  as the time of initiation of the mission, then  $d_{11}$  is the value of  $P_1[T]$  when  $P_1[0]$  is set equal to 1, and  $d_{21}$  is the value of  $P_1[T]$  when  $P_1[0]$  is set equal to zero, where  $T$  is the duration of the mission. Thus,

$$d_{11}^{(b)} = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)T}$$

$$d_{12}^{(b)} = \frac{\lambda}{\lambda + \mu} \left[ 1 - e^{-(\lambda + \mu)T} \right]$$

$$d_{21}^{(b)} = \frac{\mu}{\lambda + \mu} \left[ 1 - e^{-(\lambda + \mu)T} \right]$$

$$d_{22}^{(b)} = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)T}$$

For system c, we place the following interpretations on the d's of the matrix:

- $d_{11}$  = The expected fraction of time during the mission during which the system is operable, given that it was operable at the start of the mission
- $d_{12}$  = The expected fraction of time during the mission during which the system is failed, given that it was operable at the start of the mission
- $d_{21}$  = The expected fraction of time during the mission during which the system is operable, given that it was failed at the start of the mission
- $d_{22}$  = The expected fraction of time during the mission during which the system is failed, given that it was failed at the start of the mission.

We may calculate these probabilities directly from Equation 6. If  $P_i[t]$  is the probability that the system is in state  $i$  at time  $t$ , then the expected time  $\bar{t}$  spent in state  $i$  in an interval of length  $T$  is given by:

$$\bar{t} = \int_0^T P_i[t] dt$$

Thus, the expected fraction of time spent in that state in an interval of length  $T$  is given by:

$$\frac{\bar{t}}{T} = \frac{1}{T} \int_0^T P_i[t] dt$$

Thus  $d_{11}$  for system c is obtained from  $d_{11}$  of system b by

$$\begin{aligned} d_{11}^{(c)} &= \frac{1}{T} \int_0^T d_{11}^{(b)} dt \\ &= \frac{\mu}{\lambda + \mu} + \frac{\lambda}{(\lambda + \mu)^2 T} \left[ 1 - e^{-(\lambda + \mu)T} \right] \end{aligned}$$

and similarly:

$$d_{12}^{(c)} = 1 - d_{11}^{(c)} = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{(\lambda + \mu)^2 T} \left[ 1 - e^{-(\lambda + \mu)T} \right]$$

$$d_{21}^{(c)} = \frac{\mu}{\lambda + \mu} \left[ 1 - \frac{1}{(\lambda + \mu)T} \left[ 1 - e^{-(\lambda + \mu)T} \right] \right]$$

$$d_{22}^{(c)} = 1 - d_{21}^{(c)} = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{(\lambda + \mu)^2 T} \left[ 1 - e^{-(\lambda + \mu)T} \right]$$

#### 4.4 Capability Matrix

The element  $c_{jk}$  of a capability matrix is the  $k^{\text{th}}$  figure of merit associated with system performance in effective system state  $j$ .

The magnitude and dimensions attached to this figure of merit depend upon the specific nature of the system undergoing evaluation. For example, system a above could be an ICBM. Then  $c_j$  might be the expected number of targets destroyed, given that  $j$  ICBM's are delivered to the target area. Calculation of each  $c_j$  could, in this case, require an accounting for the targeting policy, guidance dispersion, warhead yield pattern, target area, target hardness, propellant depletion probability, enemy countermeasures, and penetration aids.

System b could be a Ground Controlled Approach (GCA) radar landing system. This system is only needed in adverse weather for aircraft returning to base. Under conditions of very low ceiling and visibility over an extensive geographical area, with the system inoperative there is a high probability that returning aircraft would be unable to land safely, and hence, would be lost. Thus, in this case, we might define our  $c_j$  as follows:

- $c_1$  = The probability that a returning aircraft would land safely, given that the GCA system is operating
- $c_2$  = The probability that a returning aircraft would land safely, given that the GCA system is inoperable.

Thus, in this case, the  $c_j$  are probabilities. Notice that the capability vector here is a measure of the degree of compatibility between two major systems, one of which is airborne, and the other of which is a ground support system.

The prime objective of the airborne system is to reach the target, destroy it, and return to the vicinity of the base; the prime objective of the ground system is to bring the aircraft in safely. Clearly, the methods presented here are not restricted to consideration of a single system.

Because the  $c_j$  depend so specifically on the type of analysis being performed, we shall defer examples of specific capability matrix development to the example which follows this discussion.

By noting the performance of this synthetic population, predictions (assessments) can be made of system effectiveness for the anticipated population.

#### 4.5 Simulation

In highly complex systems, realistic assumptions relating to the accountable factors often make the analytical formulation of Availability (A), Dependability (D), and Capability (C) matrices impractical. When this is the case, the only feasible course is to resort to simulation techniques using either analog or digital computers, or both. (References 11, 12, 13, 14)

Simulation methods available are so numerous and varied that it is impossible, here, to give a preferred method. The best method in a particular case depends on the nature of the system, the phase of the program, and the precision required. For example, in some cases it may be desirable to use simulation methods only to provide estimates of the A, D, C matrix elements; in other cases it may be preferable to by-pass the intermediate outputs and proceed directly to an overall measure of effectiveness.

Despite the possible variations, all simulation methods for estimating effectiveness have some fundamental common characteristics. First, the

relations between accountable factors and the effectiveness figure of merit must be mathematically described (References 15, 16); second, the manner in which the accountable factors may vary from one system trial to another must be known or reasonable assumptions established; and third, a large number of repeated system trials must be run on the computer using randomly selected values of the accountable factors, counting the resulting system successes and failures. This last step is commonly referred to as a Monte Carlo procedure.

Simulation techniques, like analytical methods, can be used to determine the sensitivity of the system figure of merit to variations in the accountable factors. In such an exercise deliberate (rather than random) variations are introduced in the expected values of the accountable factors and the Monte Carlo process is repeated.

In Section 5.3 the use of simulation techniques is illustrated through application to a specific effectiveness problem.

## SECTION V

### AN ILLUSTRATIVE EXAMPLE

#### 5. Introduction

In the foregoing sections the general framework of system effectiveness has been established and the tasks necessary to effectiveness prediction and analysis have been delineated. In this section the general techniques described in Section III will be illustrated in detail through application to a specific example. The example is approached first in a fundamental analytical way. This is followed by an illustration of the use of Monte Carlo simulation in the prediction and analysis.

#### 5.1 Analytical Method

The primary value of an analytical approach is that it provides an insight between system parameters and their relative impact on system effectiveness. Before discussing the details of the example, certain general comments and precautions are in order:

- (1) In selecting appropriate figures of merit, their fundamental purpose must be kept clearly in mind -- to provide management with information on which to base decisions. The numerical value of the figure of merit, when compared to quantitative system requirements, indicates whether or not corrective action is necessary. The sensitivity of the figure of merit to combinations of variations in system parameters provides a technical basis for taking corrective action.
- (2) A prediction is no better than the data on which it is based and the quality of the data is not just a matter of numerical accuracy. Specifically, data for a particular application must reflect a similarity between conditions under which the data were taken and expected operating conditions of the system under evaluation. All prediction processes require the interpolation or extrapolation of data, but the smaller



the degree of extrapolation the better the prediction will be. For example, the data sources selected for this illustration are derived from the study of ground based electronic equipment quite similar to the hypothetical system under consideration. The absence of comprehensive generic data sources is currently the weakest link in the chain of effectiveness prediction.

Proceeding with the details of the example, a very simple ground based radar system has been hypothesized for the illustration. (See Figure 3).

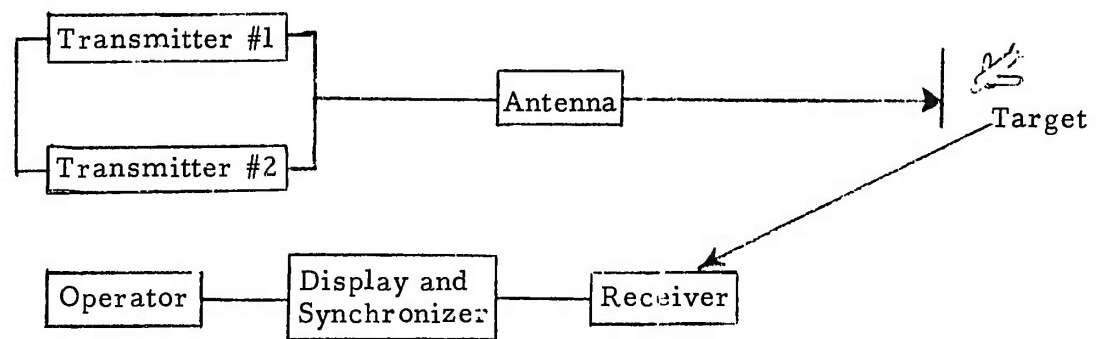


FIGURE 3

#### SCHEMATIC OF GROUND BASED RADAR SYSTEM

##### 5.1.1 Mission Definition

The system should detect target aircraft above the horizon line of sight at ranges up to 200 miles and while the target is within this maximum range, track it in range and azimuth within admissable error presenting this information to an operator at the site.

##### 5.1.2 System Description

The hypothetical system shown in Figure 3 is used for this example. It consists of two transmitters in parallel, an antenna, a receiver, a display and synchronizer, and an operator.

### 5.1.3 Specification of Figures of Merit

To show the flexibility of the basic mathematical frame work, four figures of merit are chosen:

- (1) Probability of target detection at maximum range at a random point in time.
- (2) Probability of detection at maximum range and continuous tracking within required accuracy during a prescribed time period (assumed to be 30 minutes in this example).
- (3) Probability of continuous tracking within required accuracy during prescribed time period; given successful detection.
- (4) Curve of probability to detect and track versus range (of special interest if the probability of detection at maximum range is less than the goal).

### 5.1.4 Identification of Accountable Factors

The factors to be explicitly included in the computation of the figures of merit are:

- (1) Subsystem failure rates
- (2) System and subsystem repair times
- (3) Application of maintenance manpower (unlimited resources assumed).
- (4) Repair supplies availability (unlimited resources assumed).
- (5) System configuration (redundancy and provision for emergency modes of operation).
- (6) Maximum range for successful target detection.
- (7) Radar transmitter power.

### 5.1.5 Model Construction

The first step in the analytical structuring of the model is the definition of significant states of the system, based upon possible mission outcomes. This step is followed by construction of the availability, dependability, and capability matrices, and finally the matrices are multiplied appropriately to give the required figures of merit.

#### 5.1.5.1 Definition of System States

<u>State Designator</u>	<u>Definition</u>
1	All units operating properly
2	One transmitter inoperative, the other transmitter and all other units operating properly.
3	System totally inoperative due to inoperative condition of both transmitters or any one of the other units.

#### 5.1.5.2 Constructing the Availability Vector

As defined in Section 4.2, the steady state availability vector (a single row, three column matrix in this case) is the set of probabilities that the system is in state 1, 2, or 3 at a random point in time; i.e., at some unpredictable time when a target enters its maximum range sector. Symbolically,

$$\text{Availability} = \bar{A}' = [a_1 a_2 a_3]$$

where

$a_1$  = probability that the system is in state 1

$a_2$  = probability that the system is in state 2

$a_3$  = probability that the system is in state 3

#### 5.1.5.3 Constructing the Dependability Matrix

As indicated in Section 4.3 the dependability matrix is a square array of numbers (3 rows and 3 columns in this case). Since it is assumed that no repair is possible during the critical mission time; i.e., while tracking a target, some of the elements will be zero. Symbolically,

$$[D(t = 30 \text{ min})] = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix}$$

where

$d_{11}$  = probability that the system starting the mission in state 1 completes it in state 1 (conventional reliability).

$d_{12}$  = probability that the system starting the mission in state 1 completes it in state 2.

$d_{13}$ ,  $d_{22}$ ,  $d_{23}$ ,  $d_{33}$  similarly defined.

From these definitions and the assumption of no repair during the mission it is clear, for example, that  $d_{21}$  -- the probability of starting the mission in state 2 and completing it in state 1 -- is zero.

#### 5.1.5.4 Constructing the Capability Vector

The capability of the system in this example may be defined as the probability that it can, in a given state, carry out its design functions at the required times. Since the particular mission under consideration is divided into two time sequential phases, two capability vectors will be required. They are:

$$\begin{array}{l} \text{Capability of initial target} \\ \text{detection and acquisition} \end{array} = \bar{C}(t=0) = \begin{bmatrix} C_1(0) \\ C_2(0) \\ C_3(0) \end{bmatrix}$$

where

$C_1(0)$  = probability of detection at a random time,  $t = 0$  when the system is known to be in state 1.

$C_2(0)$  = as above, but known to be in state 2.

$C_3(0)$  = as above, but known to be in state 3.

$$\text{Capability of track, given detection and acquisition} = \bar{C}(t = 30) = \begin{bmatrix} C_1(30) \\ C_2(30) \\ C_3(30) \end{bmatrix}$$

where

$C_1(30)$  = probability of track through 30 minutes, given detection at  $t = 0$  and the system in state 1 at  $t = 30$ .

$C_2(30)$  = as above, but system in state 2 at  $t = 30$ .

$C_3(30)$  = as above, but system in state 3 at  $t = 30$ .

#### 5.1.6 Data Acquisition

For this hypothetical example, data used for estimating such parameters as failure rate, repair rate, etc., are derived from sources such as those listed here:<sup>4/</sup>

- (1) MIL Handbook 217 -- electronic part reliability data from which subsystem failure characteristics can be estimated when the design is fairly complete; e. g., in the Acquisition Phase.
- (2) System Reliability Prediction by Function -- data and methods from which subsystem failure characteristics can be estimated when the design is in its preliminary states; e. g., in the Definition Phase.
- (3) Maintainability Measurement and Prediction Methods for Air Force Ground Electronic Equipment -- data and methods from which system and subsystem repair characteristics can be estimated when the design is fairly complete; e. g., in the Acquisition and Operational Phases.

#### 5.1.7 Parameter Estimation

Estimates of the parameters used to derive the basic availability and dependability factors of the effectiveness model were

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<sup>4/</sup> See Bibliography for other data sources

0 arbitrarily made using such sources as are listed above in context with the mission profile, system states and accountable factors listed in Paragraphs 5.1.2, 5.1.4, and 5.1.5. As discussed in Section 4.4, there is no general method by which elements of the capability vector can be estimated. Each individual system requires its own unique approach at each phase of system evolution. The methods used must sometimes be theoretical, sometimes empirical and sometimes a combination of the two. The example under discussion provides an excellent illustration of the manner in which a combined empirical-theoretical approach can be employed.

#### 5.1.8 Model Exercise

##### 5.1.8.1 Availability Estimates

It is assumed that all units have independent exponentially distributed failure and repair times with the following mean values:

Each Transmitter MTBF (mean-time-between-failure) = 40 hours

Each Transmitter MTTR (mean-time-to-repair) = 4 hours

Composite Antenna, Receiver, Display and Synchronizer MTBF = 100 hours

Composite Antenna, Receiver, Display and Synchronizer MTTR = 1 hour

Employing the methods of Section 4.2, the availability vector elements are now calculated to be:

$$a_1 = 0.818$$

$$a_2 = 0.149$$

$$a_3 = 0.033$$

$$\bar{A} = \begin{matrix} 0.818 & 0.149 & 0.033 \end{matrix}$$

The details of the calculation are given in

Appendix I.

### 5.1.8.2 Dependability Estimate

Continuing as in Section 4.3 and using the same assumptions regarding the units used for the availability computations we get:

$$\lambda_T = \frac{1}{(\text{MTBF})_T} = \frac{1}{40} = 0.025$$

$$\lambda_R = \frac{1}{(\text{MTBF})_R} = \frac{1}{100} = 0.010$$

where

$\lambda_T$  = each transmitter failure rate

$\lambda_R$  = composite receiver, antenna, display, synchronizer failure rate

and

$$R_T(30 \text{ min}) = \exp[-\lambda_T t] = \exp[-0.025 \times 0.5] = 0.988$$

$$R_R(30 \text{ min}) = \exp[-0.010 \times 0.5] = 0.995$$

It is now possible to compute the individual dependability matrix elements. Again the detailed calculations are included in Appendix I. The results are:

$$D(30 \text{ min}) = \begin{bmatrix} 0.971 & 0.024 & 0.005 \\ 0 & 0.982 & 0.018 \\ 0 & 0 & 1 \end{bmatrix}$$

### 5.1.8.3 Estimate of Capability

#### 5.1.8.3.1 The Elements of the $\bar{C}(0)$ Vector

The elements of the  $\bar{C}(0)$  vector represent the probabilities that the system in a given state can detect a target at maximum range. We shall assume that empirical data have been taken at the desired radar frequency with a known transmitter power and

range and with targets of typical cross sections and receivers of typical noise figures. This data then provides an estimate of the probability of detection at that known power level and range.

In Paragraph 3 of Appendix I, the following proportion is derived from theoretical considerations based upon noise theory and the radar equation:

$$\frac{\ln(1 - p_1)}{\ln(1 - p_2)} = \frac{P_{T_1} r_2^4}{P_{T_2} r_1^4}$$

where

$p_1$  = the probability of detection in case 1

$\ln(x)$  = the natural logarithm of  $x$

$P_{T_1}$  = transmitter power in case 1

$r_1$  = range in case 1

$p_2$  = the probability of detection in case 2, and so on.

For example, assume that the combined power of the two transmitters in this illustration yields, through measurement, a probability of detection at the full 200-mile range of  $C_1(0) = 0.9$ . Then, the probability of detection at half power is readily computed from the foregoing proportion to be  $C_2(0) = 0.683$ . Further, since either no power is transmitted in state 3, or the receiver chain is inoperative, the probability of detection must be zero in that state, or  $C_3(0) = 0$ .

Evidently this same relation can be used to determine capability as a continuous function of range. Used in this fashion under the conditions of this example, the relation yields the following expressions for the capability elements (see Paragraph 3 of Appendix I for mathematical details).



$$C_1(0, r) = 1 - \exp \left[ - \frac{2.3 \times (200)^4}{r^4} \right]$$

$$C_2(0, r) = 1 - \exp \left[ - \frac{2.3 \times (200)^4}{r^4} \right]$$

$$C_3(0, r) = 0 .$$

#### 5.1.8.3.2 The Elements of the $\bar{C}(30)$ Vector

The elements of the  $\bar{C}(30)$  capability vector represent the probability of tracking, with required accuracy, once the target has been detected. In the interests of simplicity, we shall assume that  $\bar{C}_1(30) = \bar{C}_2(30)$ . To facilitate a quantitative solution to the problem, we shall assume that both values are estimated to be 0.98

$$(\bar{C}_1(30) = \bar{C}_2(30) = 0.98). \text{ Of course } \bar{C}_3(30) = 0 .$$

#### 5.1.8.3.3 The Elements of the Capability Vector in the Presence of Enemy Countermeasures

From the foregoing discussions it is clear that with a little ingenuity the effects of enemy radar countermeasures could be incorporated into the model. For example, the ability to track through jamming signals may depend on the two transmitters operating at different frequencies. For each state of the system, then, there is a different probability that the system will track properly in the possible presence of jamming. To illustrate cursorily how this problem might be analyzed, let the event  $A_1$  represent successful tracking in state 1, and the event  $B$  represent the presence of jamming, with  $\bar{B}$  the absence of jamming. Then the unconditional probability of successful tracking is

$$P(A_1) = P(A_1|B)P(B) + P(A_1|\bar{B})P(\bar{B})$$

where

$P(A_1 | B)$  is the probability of successful tracking in the presence of jamming, in state 1

$P(A_1 | \bar{B})$  is the probability of successful tracking in the absence of jamming, in state 1

$P(B)$  is the probability of jamming

$P(\bar{B})$  is the probability of no jamming.

We shall not carry this part of the illustration further. It has been introduced only to indicate further possibilities of the model.

#### 5.1.8.4 Effectiveness Estimates

It is now possible to combine the foregoing matrices to establish effectiveness models for the figures of merit specified in Section 5.1.3.

$E_1$  = Effectiveness in target detection and acquisition

$$E_1 = \bar{A}' \bar{C}(0) = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} C_1(0) \\ C_2(0) \\ C_3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0.818 & 0.149 & 0.033 \end{bmatrix} \begin{bmatrix} 0.900 \\ 0.683 \\ 0 \end{bmatrix}$$

$$E_1 = 0.838 \quad .$$

NOTE: In this case the general formulation; i.e.,  $\bar{A}[D]\bar{C}$  simplifies to the above since the initial portion of the mission (detection) is considered to occur instantaneously, and consequently the dependency matrix reduces to the identity matrix,

$$[D] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$E_2$  = Effectiveness in detection and track

$$E_2 = A^T [C(0)] [D(30)] T(30)$$

where

$$[C(0)] = \begin{bmatrix} C_1(0) & 0 & 0 \\ 0 & C_2(0) & 0 \\ 0 & 0 & C_3(0) \end{bmatrix}$$

and other symbols as previously defined.

Using the numerical values previously derived:

$$\begin{aligned} E_2 &= \begin{bmatrix} 0.818 & 0.149 & 0.033 \end{bmatrix} \begin{bmatrix} 0.900 & 0 & 0 \\ 0 & 0.683 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.971 & 0.024 & 0.005 \\ 0 & 0.982 & 0.018 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.98 \\ 0.98 \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} 0.736 & 0.102 & 0 \end{bmatrix} \begin{bmatrix} 0.975 \\ 0.962 \\ 0 \end{bmatrix} \end{aligned}$$

$$E_2 = 0.824$$

$E_3$  = Effectiveness in track, given target detection and acquisition

From fundamental probability theory,

$$E_3 = \frac{E_2}{E_1}$$

Substituting the previously computed values of  $E_2$  and  $E_1$  we get

$$E_3 = \frac{0.824}{0.838} = 0.983 .$$

$E_4(r)$  = Effectiveness in detection and track versus range

$$E_4(r) = \bar{A}' [C(0, r)] [D(30)] \bar{C}(30)$$

$$= \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix} \begin{bmatrix} C_1(0, r) & 0 & 0 \\ 0 & C_2(0, r) & 0 \\ 0 & 0 & C_3(0, r) \end{bmatrix} \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix} \begin{bmatrix} C_1(30) \\ C_2(30) \\ C_3(30) \end{bmatrix}$$

We could now put in the numerical values for the elements  $a_i$ ,  $d_{ij}$ , and  $C_i(30)$  used in the other examples and for each assumed value of  $r$  compute a value of  $E_4$ . We shall not go through the details here but will show the type of results obtained in the next section of the report.

## 5.2 Application of Model Results

It was stated earlier that the purpose of a figure of merit is to provide management with information on which to make decisions. The numerical value, when compared to goals, indicates whether or not corrective action is necessary. The sensitivity of the figure of merit to variations in system parameters indicates the action or combination of actions to be taken.

And this is the real value of quantifying system effectiveness -- the ability to use the figures of merit to take corrective action. The figures of merit themselves may mean little. For example, we get 0.983. So what? The usefulness of the figure of merit comes when we compare it to some minimum acceptable requirement or other criteria, or show effectiveness variation with variations of the effectiveness parameters. Then we can make a decision. First, do we need to take action? Second, what action do we take? Let us look at some examples.

In the previous sections we computed the overall mission effectiveness ( $E_2$ ) to be 0.824. Let us assume that our original goal was 0.90 (90%). We are short of the goal, so what can be done to reach or exceed it?

First a quick glance at the different figures of merit shows that most of the trouble is in initial target detection, since the effectiveness in tracking, given target detection ( $E_3$ ) is 0.983 and effectiveness in detection alone ( $E_1$ ) is only 0.838. Further, examination of the computations used to find  $E_1$  shows that even if the availability were perfect

$$\left( \overline{A}' \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \right)$$

effectiveness would only be 0.90 since,

$$E_1' = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0.900 \\ 0.683 \\ 0 \end{bmatrix} = 0.90 .$$

Thus we must conclude that the system is limited by Capability and that either the transmitter power must be increased or range must be sacrificed.

Assume now that the system is in the Operational Phase and that an increase in power would be expensive, how much sacrifice in range is involved? Fortunately the equations can be evaluated rapidly on a computer and Figure 4 shows the overall effectiveness ( $E_2$ ) as a function of the transmitter MTBF for several values of range. The MTBF of the receiver, etc., combination and the MTTR's are considered fixed at the values previously used. From the curves we see that with a transmitter MTBF of 40 hours -- the value previously assumed -- the range sacrifice for 0.90 effectiveness is less than 20 miles; i.e., range drops from 200 to slightly over 180 miles. This might be considered satisfactory if money for improvement was hard to get.

It is noted also in looking at the curves of Figure 4 that the curves rise rather rapidly with increasing transmitter MTBF up to about 100 hours

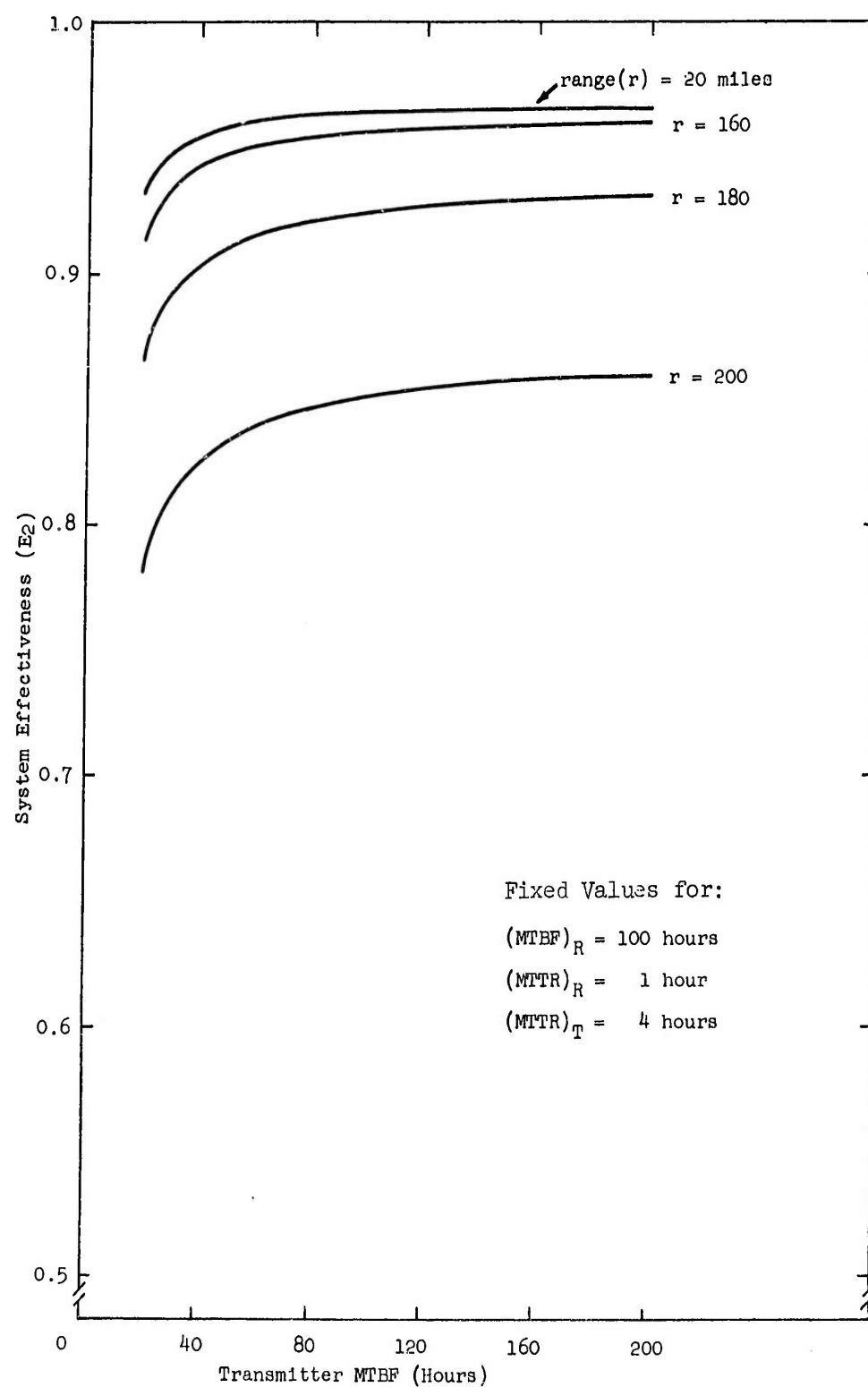


FIGURE 4. SYSTEM EFFECTIVENESS VERSUS  $(MTBF)_T$  AS A FUNCTION OF RANGE

and then increase much more slowly. It is also known that transmitter failures are expensive both in replacement parts costs and in maintenance time. It is clear that it should be possible to cut maintenance effort roughly in half by increasing this MTBF. What improvement in range for 0.90 effectiveness might be realized at the same time? Figure 5 shows overall effectiveness as a function of range for a transmitter MTBF of 100 hours. The curve crosses the 90 percent effectiveness line at almost 190 miles range. On the other hand it increases effectiveness at 200 miles range from the previous 0.824 to slightly over 0.85. The obvious question that comes to mind is: How much is this improvement in MTBF worth? If it can be done for the savings in maintenance costs alone it is evidently worthwhile. If it costs more than that one must then attempt to evaluate, also, the gain in effectiveness at fixed range or the improvement in range at fixed effectiveness. These evaluations are beyond the scope of this report, but guidelines for this purpose are the aim of Task Group IV.

Up to this point we have varied only two of the explicit "accountable factors" in our model -- the failure characteristics of the transmitter and the range requirement. It is noteworthy to see what happens when transmitter MTTR is varied. Referring to Figure 6 it can be seen that the transmitter MTTR has an appreciable effect on effectiveness only when transmitter MTBF is below about 100 hours. It can also be noted that doubling the MTBF, say from 40 hours to 80 hours has the same effect as cutting the MTTR in half. These two alternatives produce the same effect on maintenance manpower costs but not on replacement part costs. Thus in a choice between increasing MTBF or decreasing MTTR, increasing the MTBF is favored if proportionate changes can be made for the same amount of money.

Figure 6, more importantly demonstrates, as noted earlier, the overriding influence of the range requirement.

Finally, it is interesting to examine what improvement is actually realized through the use of the two paralleled transmitters. In Figure 7 the curve labeled "Configuration 1" is the system as we have been

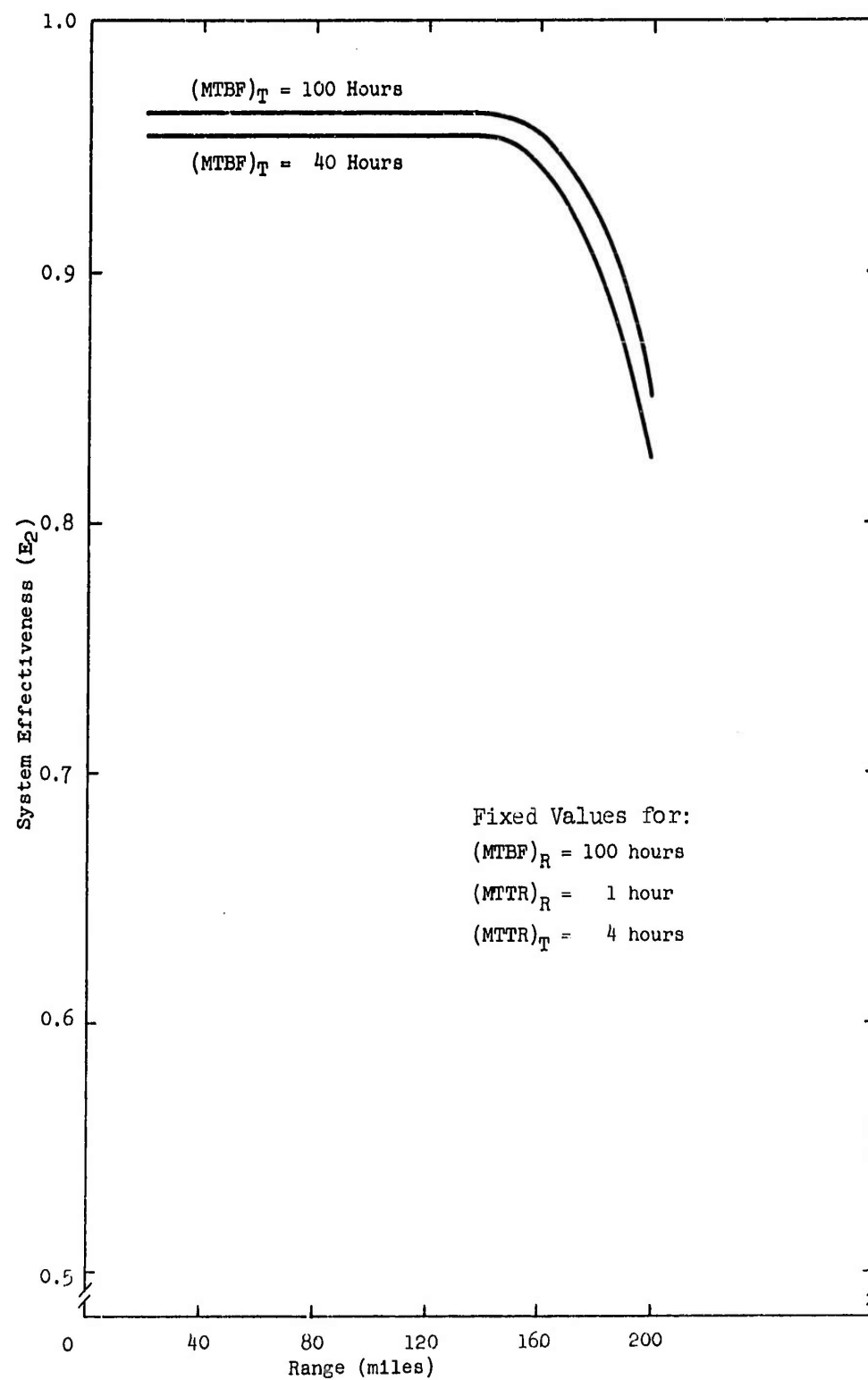


FIGURE 5. SYSTEM EFFECTIVENESS VERSUS RANGE AS A FUNCTION OF  $(MTBF)_T$



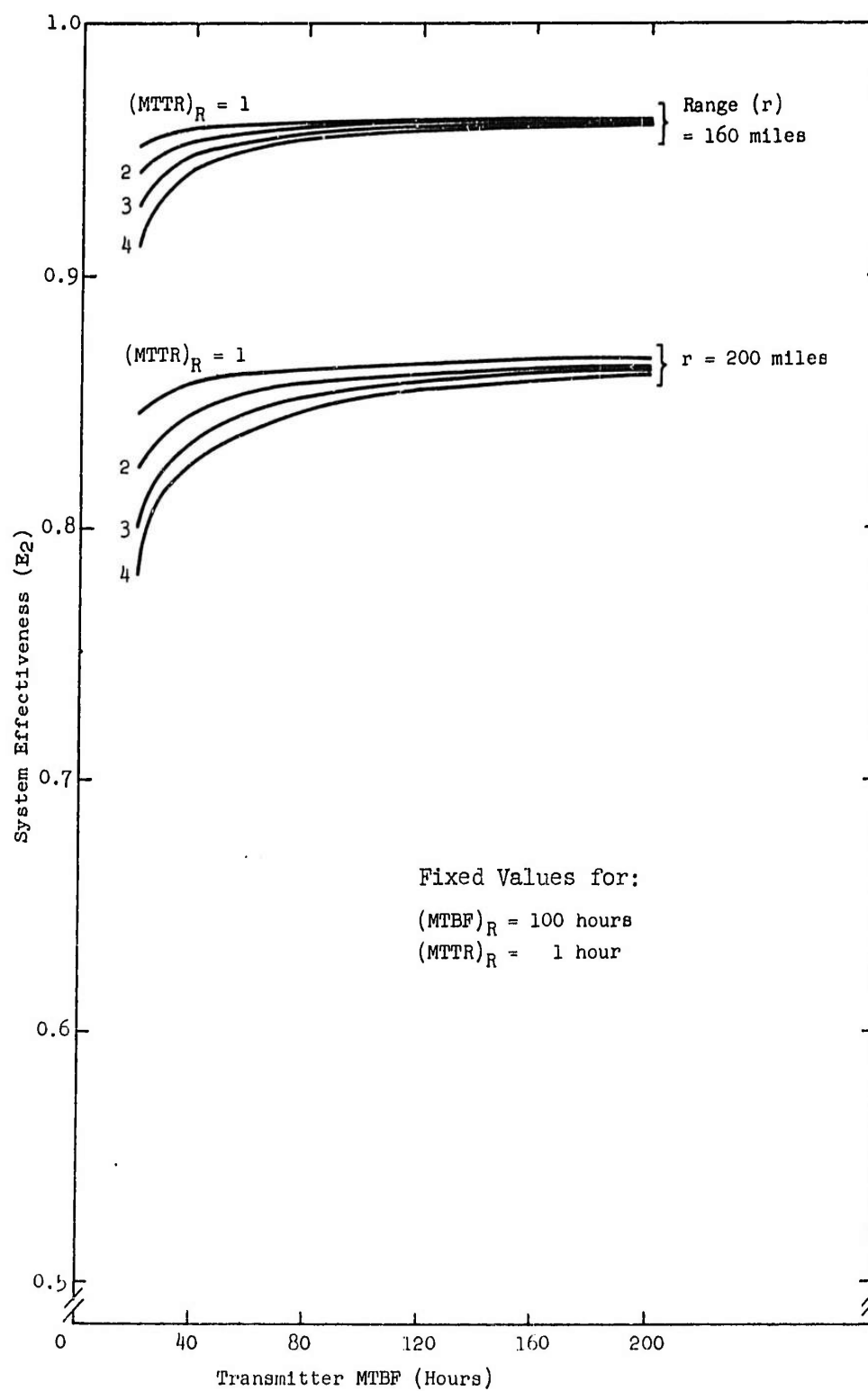


FIGURE 6. SYSTEM EFFECTIVENESS VERSUS  $(MTBF)_T$  AS A FUNCTION OF  $(MTTR)_T$  AND RANGE

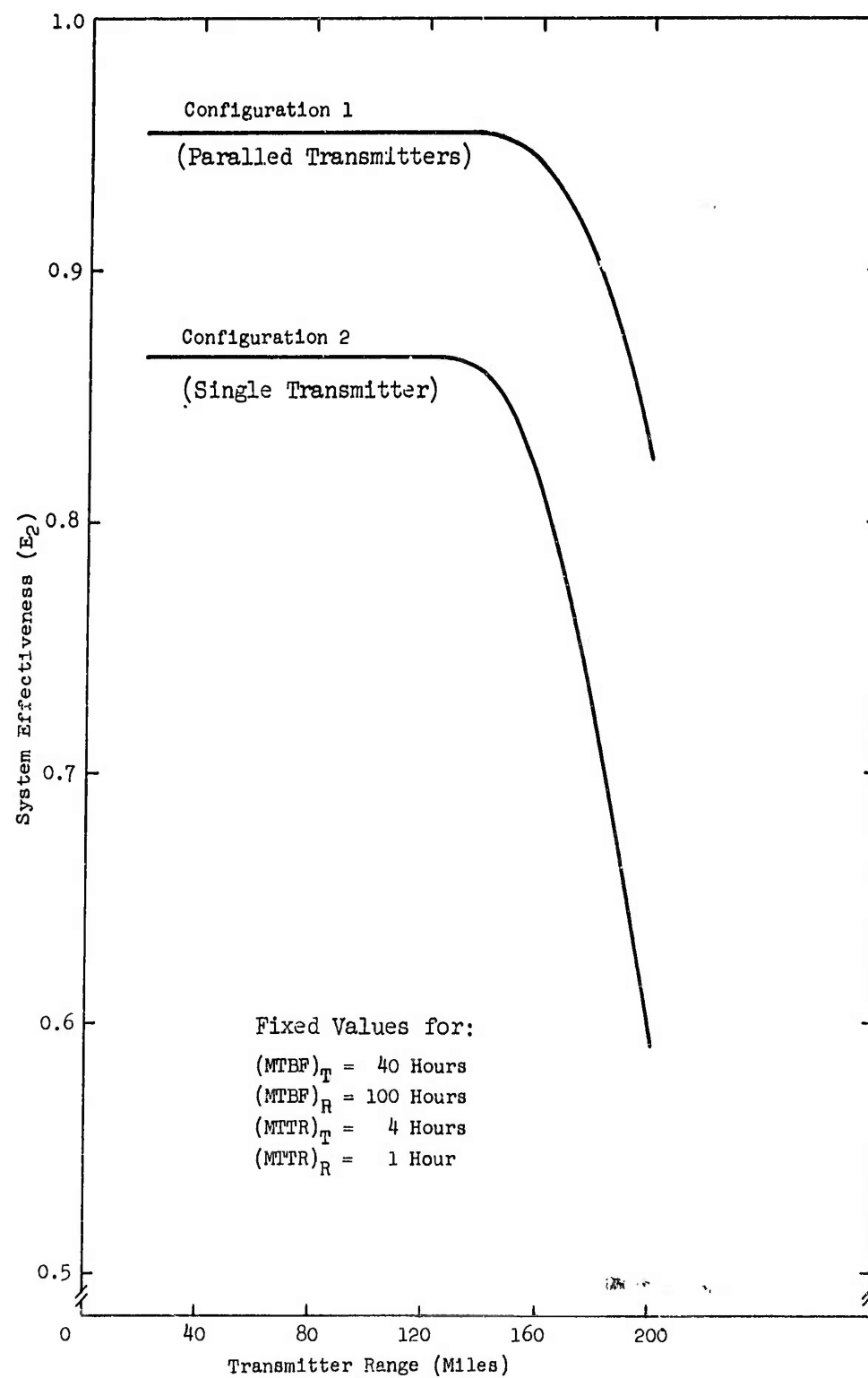


FIGURE 7. SYSTEM EFFECTIVENESS VERSUS TRANSMITTER RANGE AS A FUNCTION OF THE NUMBER OF TRANSMITTERS

discussing it; i.e., with the paralleled transmitters. The curve labelled "Configuration 2" is the system with a single transmitter. At the longer ranges the single transmitter loses effectiveness due to its lower power and the loss of redundancy. At the shorter ranges the loss is due only to the loss of redundancy. Quite evidently there is an appreciable gain in effectiveness brought about by this change in configuration. The gain however is accompanied by appreciable cost due to: (1) two transmitters instead of one, and (2) increased maintenance costs to maintain two transmitters rather than one.

Is the gain in effectiveness produced by the two transmitters worth the cost? While this question is beyond the scope of this task group effort, a brief discussion is, nevertheless, appropriate.

Especially with a defensive weapon system the "value" of the mission is essentially negative; i.e., to prevent or minimize loss. Thus it is reasonable to assume that with each failure of the system under critical use conditions some average loss, either in dollars, lives, or other measures, can be assigned. It follows then that a system that has half the failure probability of another has twice the "value," and consequently we should compare such systems on their probability of failure rather than success. This means that the complement of the effectiveness figure ( $1 - E$ ) is of greater interest for comparisons.

Following the above rationale and returning to Figure 7, we note that Configuration 2 has a probability of  $(1 - 0.59)$  or 0.41 of failure to carry out its mission if the range requirement is 200 miles and Configuration 1 has a similar probability of approximately 0.18. Thus the "value" of system 1 is

$$\frac{0.41}{0.18} \approx 2.3$$

times that of system 2. In the regions of lower range this figure rises to 3.0. Certainly then if system 2 was considered to have value commensurate with its cost, system 1 would be a better buy if the cost were less than 2.3 to 3.0

times the cost of system 2. Since the cost of the extra transmitter should not be this great and neither should the relative increase in maintenance costs, the redundancy is definitely justified.

### 5.3 Simulation Method

The application of Monte Carlo simulation methods to the previous example can be amply demonstrated by restricting attention to initial target detection at maximum range. This restriction requires only that we estimate the availability and capability vector elements; and in order to show the power of the simulation method, interdependence of system performance parameters will be taken into consideration in estimating the capability elements.

Programming a digital computer to perform Monte Carlo simulation requires three basic steps:

- (1) Mathematical formulation;
- (2) Flow diagramming; and
- (3) Coding.

Mathematical formulation suitable for digital computers requires the problem to be stated in terms of numbers and logical steps, not equations. Thus it is necessary to reduce all equations to an equivalent set of simple logical steps involving only the basic arithmetic operations of addition, subtraction, division and multiplication.

Flow diagramming consists of block diagramming, in step-by-step order, every operation the computer must perform.

Coding consists of translating the flow diagram into computer language.

In this illustration only the mathematical formulation and flow diagramming will be shown.

#### 5.3.1 Availability Vector Elements

Assuming that the time required to repair a subsystem is independent of the time-between-failures, the availability of that

subsystem can be estimated by operating it a long time and then performing the following sequence of calculations: (1) add the total time spent in an operating state; (2) add the time spent in a repair state; (3) add the times found in (1) and (2); and (4) divide the results of (1) by those of (3). The foregoing steps could be written as an equation, but as stated they are much closer to the form needed for computer simulation.

If we had actual measured numbers of the type described above we could proceed directly to an estimate of the subsystem availability. Lacking the specific numbers, they can be generated in the computer from (1) knowledge of the manner in which the time elements are distributed; (2) knowledge of the parameters of the distribution; and (3) a table of random numbers in the range 0 to 1. For example, it is commonly observed that times-between-failures are distributed exponentially and times-required-to-repair are distributed log-normally. In the first case, mean-time-between-failures is the only parameter needed to completely define the distribution. In the second case, the mean-time and some measure of dispersion are the required parameters. The tables of random numbers are universally available. Details of the computer subroutines for calculating the random time increments will not be given here since they are generally available.

Having described methods for generating times-between-failure and times-to-repair that are random in length, the detailed simulation of the system behavior can be flow diagrammed as shown in Figure 8. The detailed steps and symbols are described in TABLE III.

### 5.3.2 Capability Vector Elements

To find the probability of target detection at any range, or at a random point in time, one simulates the target, synthetically exercises the system, and notes whether the target is detected or not. The ratio of the total number of successful target detections to the total number of trials is an estimate of the probability of target detection. This probability is a function of:

- (1) The noise figure of the receiver
- (2) The power transmitted from the antenna
- (3) The performance of target detection circuits in the receiver
- (4) The operation of the display device
- (5) The competence of the human operator.

It is assumed that receiver and transmitter performance are dependent on each other; i. e., satisfactory performance can still be obtained if the transmitter power is high and the receiver sensitivity is low, or the receiver sensitivity is high and the transmitter power low.

Specifically, we assume that the ratio of receiver output signal power  $P_s$  to receiver output noise power  $P_n$  is given by:

$$\frac{P_s}{P_n} = \frac{S^2 P_R}{P_n}$$

where

$S$  = signal gain of receiver in volts output/volt input  
 $P_R$  = radar return signal power at antenna.

The radar return signal power is given by:

$$P_R = \frac{a P_T}{r^4}$$

where

$a$  = constant, involving target cross section and other factors  
 $P_T$  = total radar transmitter power  
 $r$  = range to target.

Substituting this value of  $P_R$  in the former equation gives:

$$\frac{P_s}{P_n} = \frac{a S^2 P_T}{P_n r^4} \quad (\text{Eq. 6})$$

In any given attempt to detect the target, the ratio  $P_s/P_n$  must exceed a given value  $\underline{j}$  which depends upon the target detection circuits of the receiver. Whether or not  $\underline{j}$  will be equalled or exceeded depends upon the probability distributions of transmitter power  $P(P_T)$ , receiver signal gain  $P(S)$ , and background noise power  $P(P_n)$ . The first two distributions are determined from direct measurement and observation of the transmitter and receiver. In general, they will have the form illustrated in Figures 9 and 10.

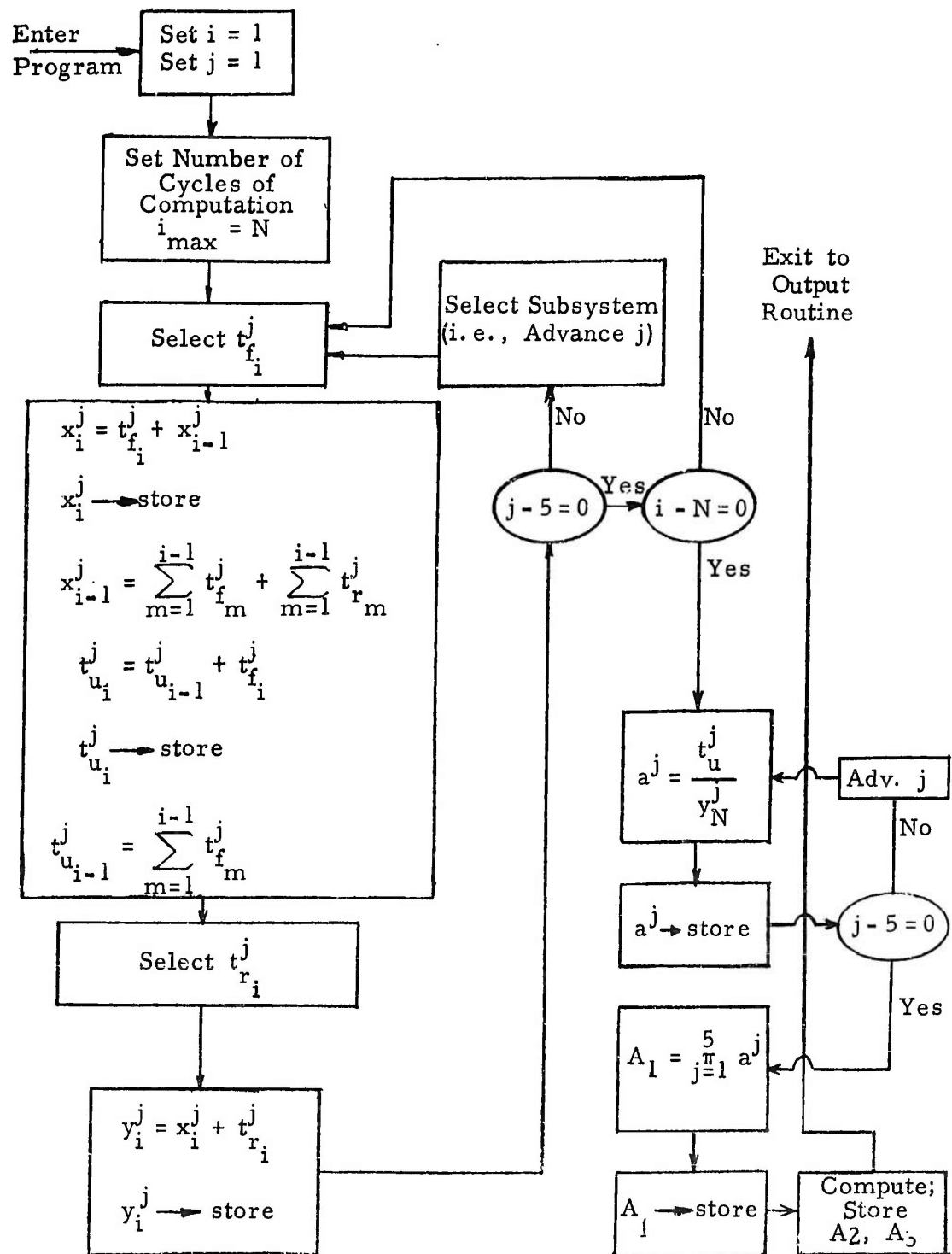


FIGURE 8

FLOW CHART FOR AVAILABILITY SIMULATION

**TABLE III**  
**STEP-BY-STEP PROGRAM**

<p> <math>j = 1 =</math> transmitter number one  <math>j = 2 =</math> transmitter number two  <math>j = 3 =</math> antenna  <math>j = 4 =</math> receiver  <math>j = 5 =</math> display. </p> <p>The subscript <math>i</math> denotes how many random times have been taken; i. e., how many cycles of calculation have been performed.</p> <p>The sequence of steps in this simulation in the <math>i^{\text{th}}</math> cycle are as follows:</p> <p>(1) Select subsystem <math>j</math></p> <p>(2) Select the <math>i^{\text{th}}</math> random time to failure for the <math>j^{\text{th}}</math> subsystem <math>t_{f_i}^j</math></p> <p>(3) Add this time to failure to the sum of all previous times to failure and the sum of all previous times to repair <math>x_{i-1}^j</math>:</p> $x_i^j = t_{f_i}^j + x_{i-1}^j$ <p>where</p> $x_{i-1}^j = \sum_{m=1}^{i-1} t_{f_m}^j + \sum_{m=1}^{i-1} t_{r_m}^j$ <p>(4) Store <math>x_i^j</math></p> <p>(5) Add this time to failure to the sum of all previous times to failure <math>t_{u_{i-1}}^j</math></p> $t_{u_i}^j = t_{u_{i-1}}^j + t_{f_i}^j$ <p>where</p> $t_{u_{i-1}}^j = \sum_{m=1}^{i-1} t_{f_m}^j$ <p>(6) Store <math>t_{u_i}^j</math></p> <p>(7) Select time to repair for <math>i^{\text{th}}</math> failure for the <math>j^{\text{th}}</math> subsystem <math>t_{r_i}^j</math>; i. e., select a random number from the random number table, enter the function generator subroutine and calculate (or look up) time corresponding to the repair rate <math>\mu^j</math> of the <math>j^{\text{th}}</math> subsystem.</p> <p>(8) Add this time to repair to the sum of all previous times to failure and the sum of all previous times to repair <math>x_i^j</math>:</p> $y_i^j = x_i^j + t_{r_i}^j$	<p>(9) Store <math>y_i^j</math></p> <p>(10) Answer the question: Is the current value of <math>j</math> equal to the total number of subsystems?</p> $j - 5 = 0$ <p>(11) If the answer to Step (10) is no, select the next subsystem; i. e., advance <math>j</math> by one and re-enter routine at Step (1).</p> <p>(12) If the answer to Step (10) is yes, set <math>j = 1</math> and answer the question: Is the current value of <math>i</math> equal to the planned number of cycles of computation <math>N</math>?</p> $i - N = 0$ <p>(13) If the answer to Step (12) is yes, enter compute availability routine (Step (15)).</p> <p>(14) If the answer to Step (12) is no, advance <math>i</math> by one and re-enter routine at Step (1).</p> <p>(15) Compute availability of <math>j^{\text{th}}</math> subsystem</p> $a^j = \frac{t_u^j}{y_i^j}$ <p>(16) Store <math>a^j</math></p> <p>(17) Answer the question: Is the current value of <math>j</math> equal to the number of subsystems?</p> $j - 5 = 0$ <p>(18) If the answer is no, advance <math>j</math> to <math>j + 1</math> and re-enter availability computation at Step (15).</p> <p>(19) If the answer is yes, compute the availability vector component <math>A_1</math></p> $A_1 = \sum_{j=1}^5 a^j$ <p>(20) Store <math>A_1</math></p> <p>(21) Compute the availability vector component <math>A_2</math></p> $A_2 = [a^1 (1-a^2) + (1-a^1) a^2] a^3 a^4 a^5$ <p>(22) Store <math>A_2</math></p> <p>(23) Compute the availability vector, component <math>A_3</math></p> $A_3 = 1 - (A_1 + A_2)$ <p>(24) Store <math>A_3</math></p> <p>(25) Exit to output routine.</p>
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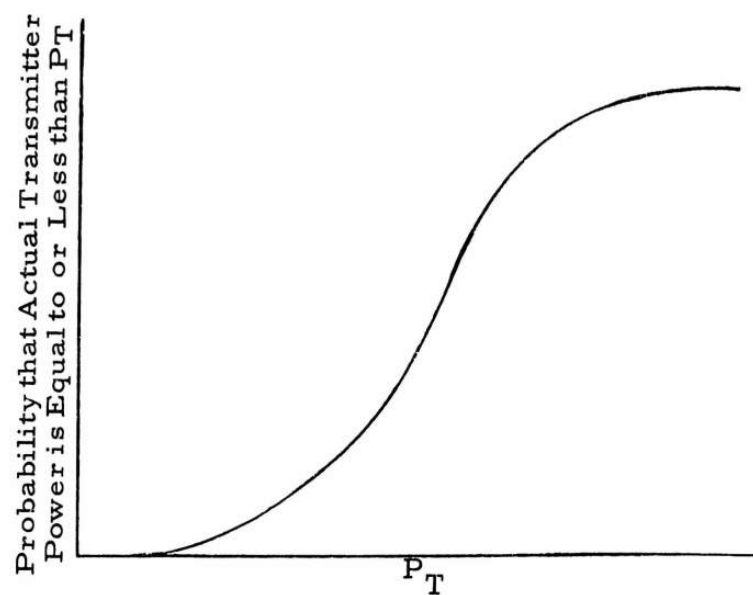


FIGURE 9

CUMULATIVE PROBABILITY DISTRIBUTION OF  
TRANSMITTER POWER VARIATIONS

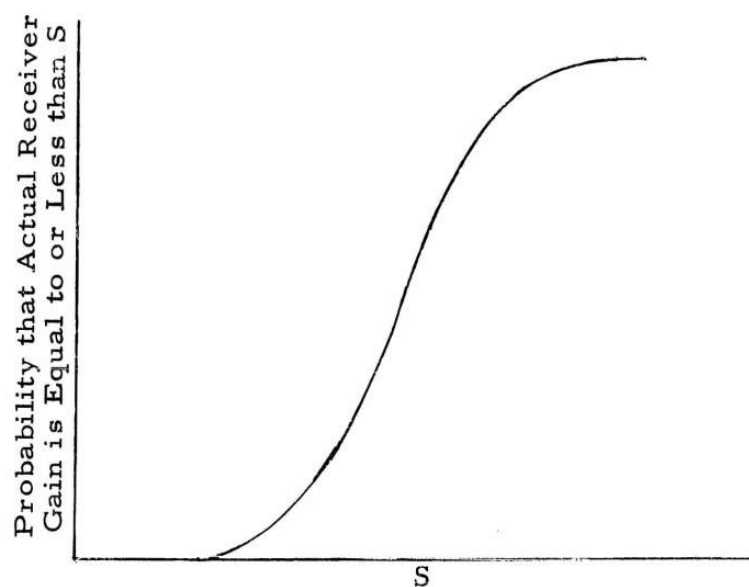


FIGURE 10

CUMULATIVE PROBABILITY DISTRIBUTION OF  
RECEIVER GAIN VARIATIONS

The probability that the noise power will be equal or less than  $P_n$  at a random point in time is also given by measurement and observation. It is given by a curve similar to that of Figure 11.

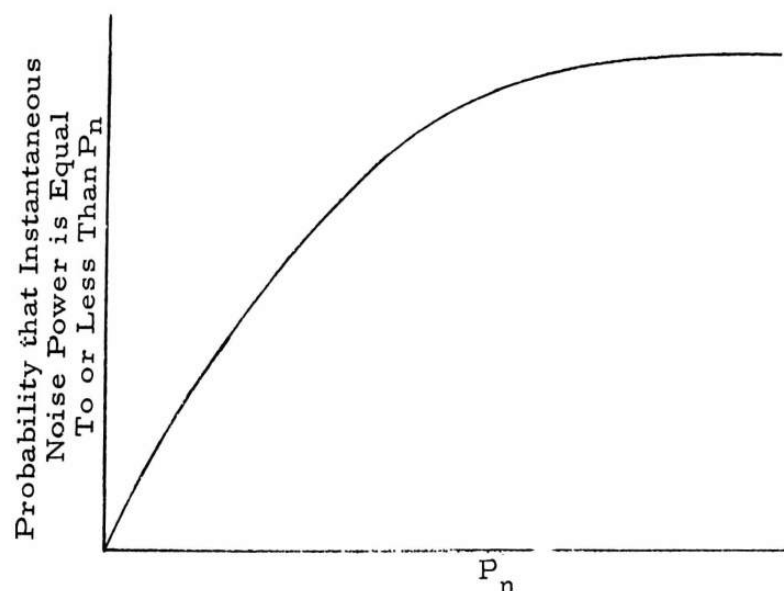


FIGURE 11

#### CUMULATIVE PROBABILITY DISTRIBUTION OF RECEIVER OUTPUT NOISE POWER

These three empirical distributions, in conjunction with Equation 6 will permit a simulation of the capability vector. The first step is to simulate random values of  $S$ ,  $P_T$ , and  $P_n$ . This is done in precisely the same manner described earlier for times to failure and times to repair. From a table of random numbers we select a value  $X_i$  which we identify as one of the probabilities  $P_i(S)$ ,  $P_i(P_T)$ , or  $P_i(P_n)$ . From the above curves we match these  $P_i$  to the corresponding signal gain  $S_i$ , transmitter power  $P_{T_i}$ , or noise power  $P_{n_i}$ , and calculate Equation 5-1. If the resultant number is equal to or greater than some preselected limit of signal to noise ratio (often taken as 1.0) we have scored a successful target detection; if not, we have scored a failure. This process of selecting the  $X_i$  and  $P_i$  is continued, remembering the successes and failures, until a satisfactory estimate of the probability of detection;  $P_d$  can be made by computing,

$$P_d = \frac{\text{total number of successes}}{\text{total number of trials}}$$

It should be noted that this simulation is done for each availability state. For state  $A_1$ , which has two transmitters in operation, we calculate

$$\frac{P_R}{P_N} = a \frac{S^2 [P_T(1) + P_T(2)]}{P_{Nr}^4}$$

For state  $A_2$ , which has only one transmitter in operation, we calculate

$$\frac{P_R}{P_N} = a \frac{S^2 P_T^{(1)}}{P_{Nr}^4}$$

or

$$\frac{P_R}{P_N} = a \frac{S^2 P_T^{(2)}}{P_{Nr}^4}$$

where  $P_T^{(1)}$  is the power transmitted from transmitter number one and  $P_T^{(2)}$  is the power transmitted from transmitter number two. Since we have not differentiated between transmitters in any way, either of the last two equations may be used for state  $A_2$ . This sequence of steps is briefly outlined in Figure 12.

### 5.3.3 Effectiveness

Since we have at this point simulated all that is required to calculate the particular figure of merit, "the probability of target detection at maximum range," we compute

$$E = A_1 C_1 + A_2 C_2; (\text{since } C_3 = 0)$$

This step is shown in Figure 12.

### 5.3.4 Simulation Accuracy

If the computer repeated the above routines an infinite number of times, the analytic results and the simulation results would be

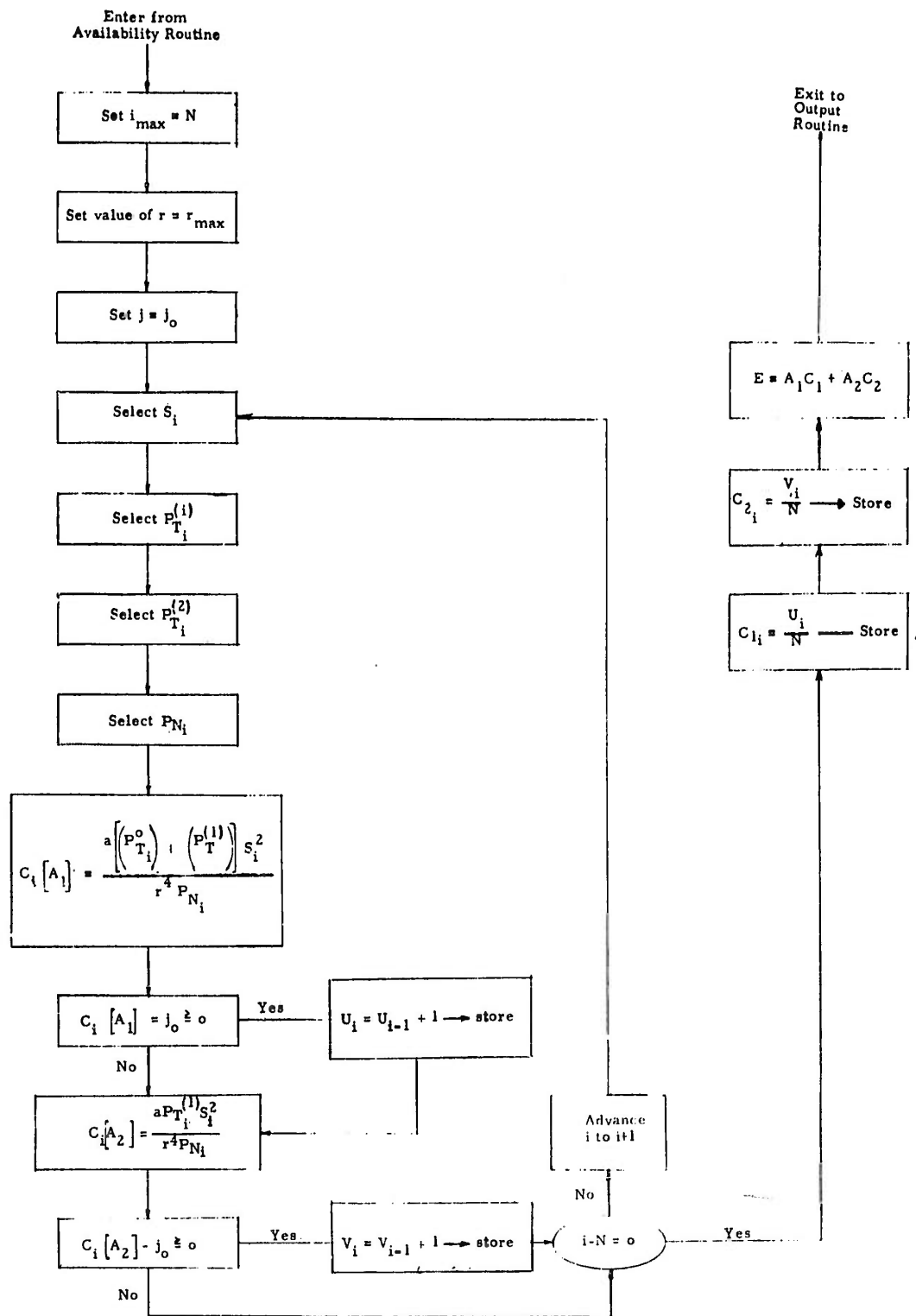


FIGURE 12. FLOW CHART FOR CAPABILITY SIMULATION

identical. However, reasonably good approximations can be obtained using small samples. For example, when the machine output is a cumulative distribution function and when the sample size increases to infinity, then with probability one, the maximum deviation between  $F_N(x)$  and  $F(x)$  tends toward zero.

$$\Pr \left\{ |F_N(x) - F(x)| > \xi \right\} \rightarrow 0 \text{ as } N \rightarrow \infty$$

$$\Pr \left\{ \sup_{-\infty < x < \infty} |F_N(x) - F(x)| > \xi \right\} \rightarrow 0 \text{ as } N \rightarrow \infty.$$

Thus, empirical distribution functions derived from a Monte Carlo simulation will uniformly approximate the shape of the true distribution function. Also, when the distribution function  $F(x)$  is a continuous function, the empirical cumulative distribution for  $N$  observations  $F_N(x)$  will uniformly approximate the true distribution with an accuracy of the order of  $1/\sqrt{N}$ , which is one consequence of the Kolmogorov-Smirnov goodness-of-fit test.

The statistic

$$D_N = \max_{-\infty < x < \infty} |F_N(x) - F(x)|$$

measures the greatest absolute difference between the true cumulative density function  $F(x)$  and the empirical cumulative density function  $F_N(x)$ .  $D_N$  is a "distribution free" statistic (i. e., its sampling distribution is independent of the underlying distribution  $F(x)$ ) and its distribution is known. This is useful in answering the following question: How large should  $N$  be chosen so that there is only a small preassigned probability that  $(D_N \geq \xi)$ ? Symbolically:

$$\Pr \{D_N \geq \epsilon\} = ?$$

If  $\alpha = 0.10, 0.05$ , and  $0.01$ , respectively, where  $\alpha$  is the acceptable risk (type I error) of test, it can be shown that the approximate sample sizes  $N$  are

$$\left(\frac{1.22}{\epsilon}\right)^2, \left(\frac{1.36}{\epsilon}\right)^2, \text{ and } \left(\frac{1.65}{\epsilon}\right)^2$$

( )  
If  $\epsilon = 0.05$  and  $\alpha = 0.10$ , then  $N = 595$ ,

If  $\epsilon = 0.05$  and  $\alpha = 0.05$ , then  $N = 740$ ,

If  $\epsilon = 0.05$  and  $\alpha = 0.01$ , then  $N = 1060$ .

## APPENDIX I

### MATHEMATICAL DETAILS OF TUTORIAL EXAMPLE

#### 1. Computation of Availability Vector Elements

Let  $\alpha_T$  = availability of each transmitter

$\alpha_R$  = availability of composite antenna, receiver, display, and synchronizer.

Then from Section 4.2:

$$\alpha_T = \frac{MTBF}{MTBF + MTTR} = \frac{40}{40 + 4} = .909$$

$$\alpha_R = \frac{100}{100 + 1} = .990.$$

Also from Section 4.2:

$$a_1 = \alpha_T \alpha_T \alpha_R = \alpha_T^2 \alpha_R = (.909)^2 .99 = .818$$

$$a_2 = [\alpha_T(1 - \alpha_T) + (1 - \alpha_T)\alpha_T] \alpha_R$$

$$= 2\alpha_T(1 - \alpha_T)\alpha_R = 2 \times .909(1 - .909).99 = .149$$

$$a_3 = 1 - (a_1 + a_2) = 1 - (.818 + .149) = .033.$$

#### 2. Computation of Dependability Matrix Elements

In Section 5.1.8.2,  $\lambda_T$  and  $\lambda_R$  were defined as the failure rates of a transmitter and the composite receiver, antenna, display and synchronizer, respectively. The corresponding unit reliabilities were given as:

$$R_T(30 \text{ min}) = \exp \{-\lambda_T t\} = \exp \{-.025 \times .5\} = .988$$

$$R_R(30 \text{ min}) = \exp \{-\lambda_R t\} = \exp \{-.010 \times .5\} = .995$$

In this example  $d_{11}$  is the probability that all units begin the mission and continue to operate throughout. Thus,

$$\begin{aligned}
 d_{11} &= R_T R_T R_R = \exp - \{ (2\lambda_T + \lambda_R)t \} \\
 &= \exp - \{ (2 \times .025 + .010)0.5 \} = 0.971
 \end{aligned}$$

Similarly  $d_{12}$  is the probability that all units begin the mission but one transmitter fails during the mission; all other units operate throughout. Thus,

$$\begin{aligned}
 d_{12} &= R_T(1-R_T)R_R + (1-R_T)R_T R_R \\
 &= 2(1-R_T)R_T R_R \\
 &= 2 \exp - \{ (\lambda_T + \lambda_R)t \} - 2 \exp - \{ (2\lambda_T + \lambda_R)t \} \\
 &= 2 \exp - \{ (.025 + .010)0.5 \} - 2 \exp - \{ (2 \times .025 + .010)0.50 \} \\
 &= 0.024.
 \end{aligned}$$

The system beginning the mission in state 1 must certainly complete it in one of the three states 1, 2, or 3 and so,

$$\begin{aligned}
 d_{13} &= 1 - (d_{11} + d_{12}) \\
 &= 1 - (0.971 + 0.024) = 0.005
 \end{aligned}$$

In similar fashion the other elements are found to be:

$$\begin{aligned}
 d_{22} &= R_T R_R = \exp - \{ (\lambda_T + \lambda_R)t \} \\
 &= \exp - \{ (.025 + .010)0.5 \} = .982
 \end{aligned}$$

$$d_{23} = 1 - d_{22} = 0.018$$

and

$$d_{33} = 1.$$



### 3. Computation of Capability Relations

In Section 5.1.8.3.1 the proportion

$$\frac{\ln(1-p_1)}{\ln(1-p_2)} = \frac{P_{T_1} r_2^4}{P_{T_2} r_1^4}$$

was presented without proof. In this expression

$p_i$  = the probability of detection in case i,

$P_{T_i}$  = transmitter power in case i, and

$r_i$  = range in case i.

The expression is derived as follows:

In the literature (Reference 17) it is shown that the probability density function of instantaneous thermal agitation noise power is:

$$f(P_n) = \frac{1}{2C^2} \exp \left\{ -\frac{P_n}{2C^2} \right\} \quad 0 < P_n < \infty$$

where

$P_n$  is instantaneous noise power

and

$C_n$  is the r. m. s. noise amplitude.

If we assume that a signal will be detected if the signal power at the receiver input  $P_R$  is equal to or greater than the noise power, then the probability of detection is

$$p(P_n \leq P_R) = \frac{1}{2C^2} \int_0^{P_R} \exp \left\{ -\frac{P_n}{2C^2} \right\} dP_n$$

which integrates to give

$$p(P_n \leq P_R) = 1 - \exp \left\{ -\frac{P_R}{2C^2} \right\}$$

or

$$\ln(1-p) = -\frac{P_R}{2C^2}.$$

From the "radar equation," it is known that the signal power returned to the receiver,  $P_R$ , is given by

$$P_R = \frac{aP_T}{r^4}$$

where

$P_T$  = transmitter power

$r$  = range

$a$  = a constant involving target cross section and other factors.

Substituting this equation in the probability equation, gives

$$\ln(1-p) = -\frac{aP_T}{2C^2 r^4}.$$

And since we assume that  $p$  has been determined for one set of values of power and range other relations can be found from the ratio,

$$\frac{\ln(1-p_1)}{\ln(1-p_2)} = \frac{P_{T_1} r_2^4}{P_{T_2} r_1^4}.$$

This expression can be solved for  $p_2$  as follows

$$\ln(1-p_2) = \frac{P_{T_2} r_1^4}{P_{T_1} r_2^4} \ln(1-p_1)$$

$$1 - p_2 = \exp \left\{ \frac{P_{T_2} r_1^4}{P_{T_1} r_2^4} \ln(1-p_1) \right\}$$

$$p_2 = 1 - \exp \left\{ \frac{P_{T_2} r_1^4}{P_{T_1} r_2^4} \ln(1-p_1) \right\}$$

If, as assumed in the example,  $p_1$  is evaluated to be 0.9 at full transmitter power output and 200 mile range, then

$$\ln(1-0.9) = -2.3$$

and

$$p_2 = 1 - \exp \left\{ -2.3 \frac{P_{T_2} (200)^4}{P_{T_1}^* r_2^4} \right\}$$

where

$P_{T_1}^*$  is the reference power.

For state 1 of the system  $P_{T_2} = P_{T_1}$  and thus,

$$C_1(0) = 1 - \exp \left\{ -2.3 \frac{(200)^4}{r_2^4} \right\}$$

In system state 2,  $P_{T_2} = \frac{1}{2} P_{T_1}$  and

$$C_2(0) = 1 - \exp \left\{ -2.3 \frac{(200)^4}{2r_2^4} \right\}$$

## APPENDIX II

### SUMMARY OF THE AIRBORNE AVIONICS SYSTEM EXAMPLE (Complete example included in Volume III.)

The purpose of this example is to demonstrate how the effectiveness evaluation techniques proposed by Task Group II and discussed earlier in this report, may be applied to the Avionics system of a tactical fighter-bomber aircraft. The example considers only the "bombing" function. Similar analyses could be made for its "fighter," "ground support," etc., functions.

It is assumed that the effectiveness evaluation is being made during the program definition phase of system life. Similar evaluations in the real world would also be necessary for system configurations established during the acquisition and operational phases. A major consideration of the program definition phase is "force structure," i. e., the number of system (aircraft) required to accomplish a specific mission. The example illustrates how the results of the effectiveness evaluation aid in making trade-off decisions.

The basic model proposed by Task Group II; i. e.,  $E = \bar{A}' [D] \bar{C}$ , is applied in this example.

#### 1. Mission Definition

The system is evaluated as a tactical weapon. The aircraft is considered to be deployed at an advanced base in the theater of operations, and is called upon to bomb tactical targets in enemy territory. It is assumed that no enemy offensive action will be mounted against the advanced base.

At any random time when an execution order is received, the aircraft shall take off immediately, receive a target assignment, proceed to target area, deliver weapon within 500 feet of target, and return to assigned operation base.

#### 2. System Description

The system consists of three major subsystems which are, where



appropriate, sub-divided into equipments.

- a. Fire control Subsystem
- b. Doppler Navigator
- c. Communication-Identification-Navigation (CIN)

Further analysis of these subsystems and their functions is made in the example. These analyses include a block diagram, time line analysis of mission, list of pertinent physical factors, etc..

### 3. Specification of Figures-of-Merit

The major figure-of-merit is the probability that the mission, as defined, will be accomplished.

In addition, the probabilities of accomplishing each of several sub-functions are regarded as appropriate figures-of-merit and are evaluated.

### 4. Identification of Accountable Factors

Accountable factors are considered in detail in the example. Major categories are:

- a. Operational conditions
- b. Support situation
- c. Data Constraints and sources.

At this early phase of program life, generic failure rate information is of prime importance since contractor bench tests from which supplementary data may be obtained, have not yet been made. Several sources for obtaining maintainability and generic failure rate information are referenced in the example.

### 5. Model Construction

#### a. Delineation of Mission Outcomes

The several possible outcomes of the bombing mission are discussed in the example; e. g. ,

- (1) Mission accomplished exactly as defined, or
- (2) Aircraft delayed in take-off, or

- (3) Aircraft does not deliver bomb within 500 feet of target.

b. Delineation of System States

The example time line diagram shows the three possible delivery modes and indicates the equipments required during various portions of the mission cycle for each mode. The probabilities of accomplishing the mission in each mode are evaluated by estimating and combining the probabilities of each sub-function; e. g. , communication, navigation, etc.. The probability of performance of each sub-function is determined through consideration of the equipment(s) which accomplish the function and their states. Only two states of each equipment are considered, viz. , operative and failed.

Some sub-functions are accomplished by redundant equipments. Therefore it is necessary to determine the probabilities of performance of these sub-functions by evaluating all combinations of redundant equipments in their respective operative and failed states.

c. System Model

The basic model expresses the system effectiveness as the sum of the products of the effectiveness values for each mode of weapon delivery and the probability of employing each mode; i. e. ,

$$E = E_1 P_1 + E_2 P_2 + E_3 P_3$$

where

$E$  = system effectiveness.

$E_1, E_2, E_3$  are effectiveness figures of modes one (1), two (2), and three (3) respectively, and

$P_1, P_2, P_3$  are the probabilities that respective mission modes will be used.

The effectiveness for each mode is expressed as the product of the effectiveness figures for the several sub-functions required during a mission employing this delivery mode; i. e. ,

$$E_1 = \pi E_i$$

where

$E_1$  = effectiveness of mission mode one (1) and

$\pi E_i$  = product of effectiveness figures of all sub-functions required for mission accomplishment in mode one(1).

The effectiveness for each sub-function is expressed as the product of availability ( $\bar{A}$ ), dependability [ $D$ ] and capability  $\bar{C}$  for each sub-function; i. e.,

$$E_i = \bar{A} [D] \bar{C}.$$

#### 6. Data Acquisition

##### a. Specify Data Elements

Elements of data from which the avionics equipment failure rates and repair rates can be computed are identified for collection.

##### b. Specify Test Methodology

Test methodology for producing failure rate and repair rate data are identified.

##### c. Specify Data Collection System

Data collection systems for obtaining both operational and test data are identified.

#### 7. Parameter Estimation

Values of individual equipments mean-time-between-failure ( $t_f$ ) and mean-time-to-repair ( $t_r$ ) are assumed for this example.

#### 8. Model Exercise

##### a. Availability

The readiness figure for each equipment is calculated from the equation,

$$P_{S_1} = \frac{t_f}{t_f + t_r}$$

where

$P_{S_1}$  is the probability of being in an operative state.

The probability that the equipment is not ready; i. e., is in state 2 (failed), is

$$P_{S_2} = 1 - P_{S_1}$$

The availability for non-redundant equipments is then:

$$\bar{A}_i' = \begin{bmatrix} P_{S_1} & P_{S_2} \end{bmatrix} \begin{bmatrix} P_{L_1} & 0 \\ 0 & P_{L_2} \end{bmatrix}$$

where

$\bar{A}_i'$  = availability vector of equipment (i)

$P_{L_1}$  = probability of aircraft launch given equipment (i) is in an operative state

$P_{L_2}$  = probability of aircraft launch given equipment (i) is in a failed state.

The availability vector for redundant equipments is:

$$\bar{A}_i' = \begin{bmatrix} P_{S_1} & P_{S_2} & \dots & P_{S_n} \end{bmatrix} \begin{bmatrix} P_{L_1} & & & 0 \\ & P_{L_2} & & \\ & & \ddots & \\ 0 & & & P_{L_n} \end{bmatrix}$$

where

$P_{L_n}$  = probability of aircraft launch given the redundant equipments required to accomplish sub-function (i) are in state  $P_{S_n}$  which is a combination of operative and failed  $P_{S_n}$  states of the redundant equipments.



b. Dependability

The dependabilities of non-redundant equipments are computed from the expression:

$$[D_i] = \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix}$$

where:

- $[D_i]$  = dependability matrix of equipment (i)
- $d_{11}$  = probability of equipment (i) completing its mission in an operative state given that it started in an operative state
- $d_{12}$  = probability of equipment (i) failing to accomplish its mission given that it started in an operative state
- $d_{21}$  = probability of equipment (i) to accomplish its mission in an operative state given that it started in a failed state
- $d_{22}$  = probability of equipment (i) failing to complete its mission given that it started in a failed state.

The dependability matrix of redundant equipments is computed from:

$$[D_i] = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & \cdot & \dots & d_{2n} \\ \cdot & & & \\ \cdot & & & \\ d_{n1} & \cdot & \dots & d_{nn} \end{bmatrix}$$

where:

- $d_{11}, d_{12}, \dots, d_{nn}$  = probability of mission accomplishment for various combinations of operative and failed states of equipments required to accomplish sub-functions (i).

c. Capability

The capability vector for non-redundant equipments is:

$$\bar{C}_i = \begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

where:

- $\bar{C}_i$  = capability vector of equipment (i)
- $C_1$  = capability of equipment (i) given it is in an operative condition (state 1)
- $C_2$  = capability of equipment (i) given it is in a failed condition (state 2).

For redundant equipments, the capability vectors are:

$$\bar{C}_i = \begin{bmatrix} C_1 \\ C_2 \\ . \\ . \\ . \\ C_n \end{bmatrix}$$

where

$C_1, C_2, \dots, C_n$  = capabilities for various combinations of operative and failed states of equipments required to accomplish sub-function (i).

#### d. Calculation of Effectiveness

The effectiveness for each sub-function is computed from the expression for the basic model; i. e. ,

$$E_i = \bar{A}_i^T [D_i] \bar{C}_i.$$

The effectiveness for each mission mode is then computed from:

$$E_{(\text{mode one})} = \pi E_{i(\text{mode one})}$$

$$E_{(\text{mode two})} = \pi E_{i(\text{mode two})}$$

$$E_{(\text{mode three})} = \pi E_{i(\text{mode three})}.$$

The single, overall system effectiveness is finally obtained from:

$$E = E_1 P_1 + E_2 P_2 + E_3 P_3.$$

### APPENDIX III

#### SUMMARY OF THE ICBM FLEET EXAMPLE (Complete example included in Volume III)

It is the specific object of this example to illustrate the analysis of an ICBM fleet in terms of the formal mathematical structure adopted by Task Group II of the Weapon System Effectiveness Industry Advisory Committee. Symbolically, this structure is given by:

$$E = \bar{A}' \begin{bmatrix} D \end{bmatrix} \bar{C}$$

where

$E$  is system effectiveness

$\bar{A}$  is the availability vector and  $\bar{A}'$  is its transpose

$\begin{bmatrix} D \end{bmatrix}$  is the dependability matrix

$\bar{C}$  is the capability (performance) vector.

In particular, the analysis illustrates the usefulness of models in assessing the impact of potential system alterations.

#### 1. Mission Definition

The general requirements of this hypothetical system may be stated as follows:

Any missile of the ICBM fleet should be ready to accept a launch directive at a random point in time, or at an arbitrary time after an alarm condition has been established at a random point in time. It should then launch successfully within a prescribed reaction time, fly a ballistic trajectory, penetrate enemy defenses, arm fuse, impact within the prescribed target area, detonate and yield as planned with a prescribed probability of target destruction.

Minimum acceptable and objective numerical system requirements for availability, countdown, flight, and probability of kill are postulated in the form of an SOR.

## 2. System Description

The system chosen for this example is an ICBM squadron consisting of nine sites with one missile per site. It is assumed that a missile contains five launch critical subsystems and a site contains three launch critical subsystems as follows:

- a. Each missile contains the following launch critical subsystems:

<u>Subsystem</u>	<u>Subsystem Designator</u>
Re-entry Vehicle	A
Guidance	B
Autopilot	C
Propulsion	D
Structure	E

- b. Each launch facility (site) contains the following launch critical subsystems:

<u>Subsystem</u>	<u>Subsystem Designator</u>
Overhead door	F
Air conditioning	G
Power generation and distribution	H

Time line analyses for each subsystem are postulated and an example time line for the checkout of an existing ICBM is given. In addition, the reliability functional block diagram of an existing ICBM is given as an illustration of the degree of system complexity that must be treated in current systems.

## 3. Specification of Figures of Merit

Several figures of merit of varying complexity are considered. The principal figure of merit is the expected number of targets destroyed per squadron calculated under the assumption that a launch directive is received at a random point in time. Subsidiary figures of merit are the relative ranking by mode of operation of subsystems in terms of availability and reliability.

#### 4. Identification of Accountable Factors

##### a. Define Level of Accountability

The degree of accountability (in this example) places the least accountable level at a subsystem, and the highest accountable level at a squadron.

The depth of detail to be accounted for in the model is specified under the following four headings, by subsystem, by site, and by squadron.

- (1) Personnel
- (2) Procedures
- (3) Hardware
- (4) Logistics.

##### (a) Personnel

The model reflects the possibility of queuing in unscheduled maintenance due to insufficient personnel.

The model does not explicitly differentiate procedural errors from human errors.

##### (b) Procedures

The model specifically accounts for the following properties of a test:

- test coverage
- test error
  - false alarm
  - oversight
- test duration
  - on alert
  - off alert

(c) Hardware

The models reflect the possibility of four failure stress levels for periodically checked subsystems depending upon the modes of operation:

- 1 alert
- 2 checkout and/or countdown
- 3 flight
- 4 demating in checkout

The model also reflects the possibility of inherently undetectable failures.

(d) Logistics

The model specifically accounts for spares provisioning under tactical launch conditions. Sparing is not treated during the pretactical situation.

b. Targeting Policy

A squadron is targeted on three objectives, three missiles to an objective.

c. Physical Factors

(1) The launch site is regarded to be impervious to countermeasures except when the overhead door is open. (Consider ground invulnerability to be unity.)

(2) For the class of target considered, the warhead exhibits a unity damage function.

(3) The cross range and down range miss distances arising from errors of the guidance system are normally distributed and independent.

(4) The probability of propellant depletion is zero for the target ranges used.

(5) Under tactical launch conditions two launch attempts may be made, since each site stocks sufficient spares to repair one countdown

abort. No retargeting capability exists.

(6) The reliability and performance capability of the communication system is unity.

(7) Penetration probability is unity.

d. Personnel Composition

Each squadron is supported by four maintenance crews. A crew works an eight-hour shift with every fourth day off. During emergency conditions not lasting longer than one week, all crews may be put on twelve-hour duty, two crews operating simultaneously. Maintenance equipment is redundant to this extent. A full crew is required to maintain, checkout, and/or repair a failed missile or launch facility. Scheduled maintenance does not create queuing problems.

Each launch site is fully manned twenty-four hours a day.

5. Model Construction

a. Delineation of Mission Outcomes

The following mission outcomes are treated in the model.

(1) Total failure (full target survival)

- (a) Not ready to enter countdown.
- (b) Aborts countdown.
- (c) Catastrophic failure in flight.
- (d) No yield.

(2) Partial failure (or success): (incomplete target destruction)

- (a) Falls wide of target with proper yield.
- (b) Falls on target with low yield.

(3) Total success (target destroyed)

b. Delineation of System States

During pretactical conditions each of the subsystems is assumed to occupy any one of the following seven states:

- (1) "up" (on alert) and nonfailed
- (2) "up" and failed detectably
- (3) "up" and failed undetectably
- (4) "down" (in checkout/repair) and nonfailed
- (5) "down" and failed detectably
- (6) "down" and failed undetectably
- (7) "down" and in rework - Time Compliance Technical Order (TCTO).

In addition, one additional state is considered in the post alarm environment namely, down and awaiting repair (queuing).

c. System Model

The principal model is

$$E = \bar{A}' [D] \bar{C}$$

where

$E$  = expected number of targets destroyed per squadron.

$\bar{A}'$  = availability vector

= probability that at a random point in time exactly  $k$  of the 9 sites is up and nonfailed where  $k$  ranges from zero to 9 inclusive

$[D]$  = dependability matrix  
 = array of transition probabilities giving the probabilities that if  $k$  of the 9 sites are available,  $r$  of them will launch, fly, impact in the target area, and detonate as planned; where  $r$  ranges from zero to  $k$  inclusive and  $k$  ranges from zero to 9 inclusive

$\bar{C}$  = capability vector

= probability that a total of  $y$  targets will be destroyed; given that  $r$  missiles are delivered to the target area and are detonated.

d. Subsystem Models

Subsystem models for availability and reliability are developed for each of the eight launch critical subsystems. These models illustrate both periodic checkout and continuous monitoring maintenance policies.



( ) Dependent checkout policies are considered, as well as the properties of test coverage and accuracy.

e. Piece Part Models

A detailed model is developed for the re-entry vehicle illustrating model development at a level less than the subsystem level.

6. Data Acquisition

a. Specify Data Elements

The elements of data required to estimate the model parameters are listed and compared to the elements of data currently acquired in the SAC U-82 and U-86, and the AFLC AFM 66-1 data collection system. Deficiencies are noted.

b. Specify Test Methodology

Test methodology is discussed, involving:

- (1) normal field operation
- (2) special field exercises
- (3) special nonfield tests
- (4) failure analysis
- (5) depot data

c. Specify Data Collection Systems

As cited above, the SAC U-82 and U-86, and AFLC AFM 66-1 data systems are discussed.

7. Parameter Estimation

A series of maximum likelihood and least squares estimators are developed in terms of the type of data available. Attention is given to the use of both field data and the results of tear down failure analyses.

8. Model Exercise

Point estimates of the availability, reliability (dependability), and performance (capability) of each subsystem are made. These estimates are combined to obtain  $\bar{A}$ ,  $[D]$ , and  $\bar{C}$ . Finally, these measures are

combined to obtain a point estimate of E, the expected number of targets destroyed per squadron.

#### 4. Applications

##### a. Comparative Systems Analysis

The results of the model exercise are compared to the SOR. This comparison indicates that the minimum acceptable values for system reliability in countdown and flight are met, although the reliability of the re-entry vehicle is clearly susceptible to improvement. The true availability of the system is woefully lower than the acceptable minimum, although the apparent availability is relatively high. The per-unit kill probability is also in drastic need of improvement.

##### b. Parameter Variation Studies

Parameter variation studies are initiated on the availability and capability factors to assess the potential for system improvement. It is shown that:

- (1) improved monitoring and increased reliability of the power generation and distribution subsystem in conjunction with
- (2) a drastic shortening of the times between scheduled checkouts on several subsystems

and

- (3) an increase in guidance accuracy by a factor of two will be required to achieve minimum acceptable system performance.

The question of costs, schedules, confidence factors, relative strategic value of the system, and technical feasibility of accomplishing the required system alterations are not considered.

## APPENDIX IV

### SUMMARY OF THE RADAR SURVEILLANCE SYSTEM EXAMPLE (Complete example included in Volume III.)

This example illustrates for this type of defense system the effectiveness prediction techniques discussed earlier in this document. The tasks required to evaluate system effectiveness have been considered throughout the four phases of system life, and the increasing amount of detail which is necessary as the system evolves is shown. A particular feature of this example is an illustration of the manner in which the total number of system states is reducible to a lesser number of significant states.

#### 1. Mission Definition

The requirements of this fictitious system are:

- a. Detect airborne objects in the surveillance sector at a range of not less than 3,000 nautical miles.
- b. Identify the objects, and determine, within 30 minutes whether or not they constitute a threat.

#### 2. System Description

As the system is shown to evolve through the four program phases, its configuration changes with design improvements. For comparative purposes, two models are constructed and carried through the example. The first depicts system configuration during the program definition phase. The second depicts system configuration at the end of the acquisition phase and beginning of the operational phase. The model pertaining to the acquisition phase configuration is omitted for the sake of brevity.

##### a. Program Definition Phase

It is decided during the program definition phase that three radar equipments will be required, each providing surveillance of a specified sector. Associated functions necessary to complete the surveillance requirement are identified as follows:

- (1) A data link function for each radar equipment to transfer radar data to a computational center;
- (2) A computer function to store input data and to predict impact areas; (the single computer shall serve all radar equipments);
- (3) Three communication functions, each of which shall convey data from its associated radar to its data processor;
- (4) Three data processors and three displays to present data from associated radar to decision maker; and,
- (5) Necessary prime power to support each of the three subsystems.

b. Operational Phase

At this point of system life the example system configuration is firmed-up and is now defined to be:

- (1) Three radar equipments, each of which shall provide surveillance of a selected sector. Any of the radar equipments shall be capable of operating with any of the three antennas. Switching shall be possible in less than three minutes for the transmitters and in less than 1.5 minutes for the receivers;  
  
A spare transmitter shall be provided which can be switched into any of the three equipments in less than three minutes.
- (2) Two data link subsystems, either of which shall be completely capable of handling all radar data.
- (3) Two storage and computing subsystems, either of which shall be completely capable of storing all input data and predicting impact area.
- (4) Three communications subsystems, any one of which shall be completely capable of conveying all necessary data to the decision point.
- (5) Two data processor subsystems, either of which shall be completely capable of processing all required data; Three data display subsystems, any one of which shall be completely capable of displaying all required data.
- (6) Six independent power generating devices, any four of which, when operating at full capacity, shall be capable of supplying the total power requirement. In normal operations, five generators shall be on-line, each

operating at 80 percent of full load capacity. Power lines capable of transferring power with no more than 0.5 percent power loss at maximum load.

c. Block Diagrams

Two functional block diagrams representing the system as described during the definition and operational phases are included in the example.

d. Mission Profile

The equipment is run continuously until failure.

3. Specification of Figure(s) of Merit

The figure of merit for this system was taken to be "the probability that the system will provide a 30 minute warning, given an enemy airborne pre-emptive attack at a random point in time."

4. Identification of Accountable Factors

The example includes a list of accountable factors. The list is not necessarily exhaustive but is representative of those requiring consideration. Some of the example factors are listed below:

- a. Atmospheric phenomena
- b. Enemy actions and countermeasures
- c. System hardware
- d. Maintenance concept
- e. Data constraints
- f. Data sources.

Publications which supply generic failure rate and maintainability information are listed. These must be supplemented by contractors' bench test data.

5. Model Construction

a. Delineation of Mission Outcomes

In the example, consideration is given to the possibility that a warning of an attack will be given even though some of the radar equipments

are in a failed condition. Therefore an evaluation of system capabilities in various system states was made.

b. Delineation of System States

Only "significant" system states were considered in the evaluation of the two system configurations (1-definition phase, and 2-operational phase). The total number of system states evaluated was reduced to only those having an effect on system capability. These were called "significant" states. A further simplification was made by treating collectively as a single state all states in which no system capability exists.

Diagrams of system states for the two system configurations described are included in the example. Four (4) system states were considered to be significant for the definition phase configuration and seventeen (17) for the operational phase configuration.

c. System Model

The basic model employed for both system configurations is:

$$E = \bar{A}' [D] \bar{C}$$

where

$E$  = probability that the system will provide a 30-minute warning, given an enemy airborne pre-emptive attack at any random point in time;

$\bar{A}$  = availability vector;  $\bar{A}'$  is its transpose  
= probability that at any random point in time, the system will be in state  $i$ , where  $i$  can be any integer from 1 to  $n$ , inclusive,  $n$  = number of system states to be considered;

$[D]$  = reliability (dependability) matrix  
= probability of transition from system state  $i$  to system state  $j$  during the required operating period (0.5 hours), given state  $i$  at the beginning of this period;

$\bar{C}$  = capability vector  
= probability that the system can successfully perform the required functions, given that the system is in state  $j$  during the period of interest.

Equations are developed for determination of the components of  $\bar{A}$ ,  $[D]$  and  $\bar{C}$  corresponding to the various system states. These equations are listed for the system configurations considered.

## 6. Data Acquisition

Data from which the radar equipments failure rates and repair rates can be computed are identified.

## 7. Parameter Estimation

The methodology for estimating mean-time-to-failure ( $t_f$ ) and mean-repair-time ( $t_r$ ) is discussed in the example. References to publications containing prediction models for estimating mean-repair-times are given.

Again, for the sake of brevity, and because of the complexities involved in estimating  $t_f$  and  $t_r$  "guesstimates" are made of  $t_f$  and  $t_r$  numbers for the various radar equipments. These are listed in a table.

## 8. Model Exercise

### a. Availability

The availability of each subsystem is determined by application of  $t_f$  and  $t_r$  numbers to equation

$$A_I = \frac{t_{fI}}{t_{fI} + t_{rI}} .$$

These results are then used to determine the components of the availability vector.

### b. Dependability

The dependability matrix  $[D]$  is determined for the four system states of the definition phase system configuration in order to illustrate this computation. However it is assumed to be equal to one (1) for the operational system configuration because of the relatively short duration of the mission time. This useful assumption of course greatly simplifies the effectiveness calculations.

c. Capability

Numerical values for each component of the capability vector were assumed for both the definition phase and operational phase configurations. Each assumed value represents the probability of successful mission accomplishment for a corresponding system state.

d. Effectiveness

After determination of the availability and capability vectors and the dependability matrix, the effectiveness (E) is determined for both configurations by multiplication of these three terms. Thus:

$$E = \bar{A}' [D] \bar{C} \quad .$$



## APPENDIX V

### SUMMARY OF THE SPACECRAFT SYSTEM EXAMPLE (Complete example included in Volume III.)

This example illustrates in some detail a method by which Dependability of a spacecraft may be determined from conservative estimates of hardware reliability. This approach is suggested in the absence of large amounts of test data on the vehicle being evaluated. This usually occurs during the program definition and early system acquisition phases of programs on substantially new systems. It is also useful for evaluating extremely costly systems of which only a few are to be constructed. No effort will be made in this example to treat Availability or Capability, beyond illustrating their tie-in with Dependability to calculate Effectiveness; i.e.,

$$E = A' [D] C .$$

The assumed purpose of this evaluation is the determination of critical elements in the proposed spacecraft configuration.

#### 1. Mission Definition

The spacecraft system shall be capable of placing a variety of payloads including multiple satellites into precise orbits about the earth. It shall have the capability of restarting in space after a sufficient coast period dependent on the specific payload and attitude orientation in space. The system shall be designed as an upper stage rocket propulsion vehicle.

#### 2. System Description

The system described herein is a spacecraft for placing satellites in earth orbits. The spacecraft is a liquid-propellant upper-stage rocket propulsion vehicle providing all the control elements necessary for placing a variety of payloads in precise orbits about the earth.

The spacecraft is composed of four major subsystems, namely, propulsion, forward section, ordnance (except range safety) and structure.

Further analysis of each subsystem and its function is made in the example. Included are block diagrams, mission sequences, environments, etc..

### 3. Specification of Figure of Merit

The system effectiveness is evaluated in terms of the probability that the system will achieve a given mission. This example concentrates on the determination of the dependability matrix in the effectiveness model

$$E = \bar{A}' [D] \bar{C}$$

where

$\bar{A}'$  is a 1 x 3 row vector comprising the relative levels of, or probabilities of availability in, each of three states of readiness which are not, themselves, calculated in this example

$[D]$  is a 3 x 3 dependability matrix comprising transition probabilities of performing (i. e., completing a mission) in each of three system states depending on the initial state, or readiness

$\bar{C}$  is a 3 x 1 column vector of the relative payoffs, or success probabilities, associated with each state of system performance -- again, not calculated for this example.

### 4. Identification of Accountable Factors

The following items are quantitatively considered in evaluating the system effectiveness for the upper-stage spacecraft.

- a. The mean time to failure
- b. The mission length
- c. The structural integrity
- d. The maintenance policy
- e. The system capability.

### 5. Model Construction

#### a. Delineation of Mission Outcomes

There are three possible mission outcomes:

- (1) perfect operation -- in desired orbit with all data
- (2) imperfect, but acceptable operation -- in orbit, partial data
- (3) unacceptable operation -- no orbit, no data.

b. Delineation of System States

Three states are defined in accordance with the specified mission outcomes:

<u>State Designation</u>	<u>Equipment Condition</u>
1	perfect operation
2	imperfect, but acceptable operation
3	unacceptable operation

c. Models

In the example, basic reliability models from which the dependability matrix is developed, are described.

Consideration is given to the probability that all parts will function properly and to the probability that all essential parts will function properly. Both time-stress and peak-stress analyses are demonstrated, as are methods for adjusting for environmental and application conditions. Further, an analysis of the reliability of structural components is made.

These analyses are employed in developing the dependability matrix of the following form:

$$[D] = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & d_{22} & d_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

where

$d_{11}$  = probability that spacecraft will have no failure during mission; given that it is initially non-failed

$d_{12}$  = probability that the spacecraft will have one or more non-critical failures in the mission, given that it was initially non-failed

$d_{13}$  = probability that the spacecraft will have one or more critical failures in the mission; given that it was initially non-failed

$d_{22}$  = probability that the spacecraft will have a critical failure in the mission; given that it has initially one or more non-critical failures

$d_{23}$  = probability that the spacecraft will have one or more critical failures in the mission; given that it initially has one or more non-critical failures.

## 6. Data Acquisition

(See parameter estimation.)

## 7. Parameter Estimation

The subsystem failure rates shown in the example for the various mission segments, are derived from specific component failure rates which are reciprocals of component mean-time-to-failure as estimated from industry experience and testing.

Component failure rates are adjusted for environmental stress and for condition of use or application, then combined into assembly and subsystem failure rates. Since component data were obtained from various industry and test sources the adjustment factors included escalation factors from one source, such as generic industry lists to another source, such as hangar check-out data. The joint effects of all these adjustments are reflected in K-factors by which failure rates from the various sources are multiplied to obtain rates applicable to various aspects of the mission.

Since the failure rates are applicable to specific aspects of the mission, various segments of the mission are treated individually.

Consideration is also given to those failures which are not random with respect to time. This occurs when peak stress during mission operation exceeds the strength of the part. Probability statements that this kind of failure will not happen may be termed peak stress reliability and are included in the example.

Included (without specific modeling or calculation) in the capability vector are the concepts of expected return (or payoff) from perfect, acceptable or unacceptable payload injection into orbit, respectively, weighted for the reliability of the payloads and of the externally furnished command guidance subsystem.

### 8. Model Exercise

The dependability matrix  $[D]$ , is calculated in this example to be

$$[D] = \begin{bmatrix} d_{11} & d_{12} & d_{13} \\ 0 & 0 & d_{23} \\ 0 & 0 & d_{33} \end{bmatrix}$$

where the subscripts refer to the functional states

- 1     Perfect
- 2     Imperfect but acceptable
- 3     Unacceptable,

and the double subscript,  $ij$ , denotes the probability of entering state  $j$  when the initial state of readiness is state  $i$ .

Calculations reflecting assumed values for Availability and Capability are included in the example.

## REFERENCES

1. H. S. Balaban and D. L. Costello, "System Effectiveness Concepts and Analytical Techniques," ARINC Research Publication No. 267-01-7-419, ASD/AFSC Contract No. AF 33(657)-10594, ARINC Research Corporation, Washington, D. C., January 1964
2. G. H. Sandler, System Reliability Engineering, New Jersey, Prentice-Hall, 1963
3. Sylvania Electronic Defense Laboratories, "Mathematical Models for System Reliability," SEDL Publication No. EDL-E35, DDC No. AD 228131, August 1959
4. J. D. Hosford, "Measures of Dependability," Journal of Operations Research Society of America, Vol. 8, No. 1, p. 53, January-February 1960
5. RCA Service Company, "Maintainability Prediction Techniques," RADC Document No. RADC-TDR-62-156 (Phase IV), RADC Contract No. AF 30(602)-2057, DDC No. AD 277199, 15 March 1962
- 6a. E. L. Kaplan and P. Meier, "Nonparametric Estimation from Incomplete Observations," (Bell Laboratories Monograph 3160) Jour. of Am. Stat. Assoc. 53, No. 282, pp. 457-81, June 1958
- 6b. B. Epstein and M. Sobel, "Life-Testing," Jour. of the Amer. Stat. Assoc. 48, pp. 486-501, 1953
- 6c. B. Epstein and M. Sobel, "Some Theorems Relevant to Life Testing from an Exponential Distribution," Annals of Math. Stat. 25, pp. 373-81, 1954
- 6d. B. Epstein and M. Sobel, "Sequential Life Tests in the Exponential Case," Annals of Math. Stat. 26, pp. 82-93, 1954
- 6e. B. Epstein, "Truncated Life Tests in the Exponential Case," Annals of Math. Stat. 25, pp. 555-64, 1954
- 6f. W. Weibull, "A Statistical Distribution Function of Wide Applicability," Jour. of Appl. Mech. (ASME) 18, pp. 293-97, 1951
- 6g. J. H. K. Kao, "A Graphical Estimation of Mixed Weibull Parameters in Life Testing Electron Tubes," Technometrics 1, Part 4, pp. 389-407, 1959
- 6h. D. A. Darling, "The Kolmogorov-Smirnov, Cramer-Van Mises Tests," Annals of Math. Stat. 29, pp. 823-38, 1958

- 6i. W. G. Cochran, "The Chi Squared Test of Goodness-of-Fit," *Annals of Math. Stat.* 23, pp. 315-45, 1952
- 6j. D. R. Cox, "Some Statistical Methods Connected with Series of Events," *J. of Royal Stat. Soc.* 17, Series B, No. 2, pp. 129-64, 1955
- 6k. A. Wald and J. Wolfowitz, "An Exact Test of Randomness in the Non-Parametric Case Based on Serial Correlation," *Annals of Math. Stat.* 14, pp. 378-88, 1943
- 6l. B. Epstein, "Tests for the Validity of the Assumption that the Underlying Distribution of Life is Exponential, I," *Technometrics*, Vol. 2, No. 1, 1960
- 6m. H. Raiffa and P. Schlaifer, Applied Statistical Decision Theory, Chapter 10, Harvard University, Boston, 1961
- 6n. J. R. Rosenblatt, "Confidence Limits for the Reliability of Complex Systems," Statistical Theory of Reliability, edited by M. Zelen, University of Wisconsin Press, Madison, 1963
- 6o. N. Kaufman, "Weapon System Availability Models and the Estimation of their Parameters," ASQC Long Island Quality Control and Reliability Conference, 20 April 1963
- 7. I. Doshay, et al, "Development of an Analytical Model for ERIE," Doc. RTD-TRD-63-4101, AF Contract No. AF 33(657)-9949, Space-General Corporation, January 1964
- 8. P. M. Morse, "Queues, Inventories and Maintenance," John Wiley and Sons, New York, 1958
- 9. RCA Service Co., "Guide for the Preparation of TDP Dependability Plans," prepared for Fleet Electronics Effectiveness Branch, Code 679C, BuShips, 2 March 1964
- 10a. J. J. Coleman and I. J. Abrams, "Mathematical Models for Operational Readiness," *Journal of Operations Res. Society of America*, Vol. 10, No. 1, p. 126, January-February 1962
- 10b. A. J. Monroe, "Weapon System Capability: Availability Models and Parameter Estimation," *Proceedings of the Aerospace Reliability and Maintainability Conference*, Washington, D. C., May 6-8, 1963
- 11. H. Kahn, "Applications of Monte Carlo," Document No. RM-1237 AEC, USAEC Contract No. AT (11-1)-135, 19 April 1954

12. H. Kahn and A. W. Marshall, "Methods of Reducing Sample Size in Monte Carlo Computations," Jour. of ORSA, Vol. 1, No. 5, November 1953
13. H. Goode and R. Machol, Control Systems Engineering, "Simulation," McGraw-Hill, Chapter 26, 1957
14. I. Bosinoff, et al, "Mathematical Simulation and Reliability Prediction Report," Sylvania Doc. No. F 491-1, DDC Document No. AD 271367, Contract No. AF 30(602)-2376, Sylvania Electronic Systems Div., October 1961
15. I. Bosinoff, et al, "Transfer Functions in Mathematical Simulation for Reliability Prediction," Sylvania Document No. F 491-2, Contract No. AF 30(602)-2376, Sylvania Electronic Systems Div., 30 January 1963
16. J. Kielson and A. Kooharian, "On Time Dependent Queuing Processes," Annals of Mathematical Statistics, March 1960
17. Dover Publications, Selected Papers on Noise and Stochastic Processes, edited by Nelson Wax, p. 230, Eq. 3.9-17, New York, 1954



## BIBLIOGRAPHY

H. Balaban and A. Drumm, "Prediction of Field Reliability for Airborne Electronic Systems," ARINC Research Publication No. 203-1-344, Contract No. AF33(657)-7382, ARINC Research Corp., 31 December 1962.

G. J. Blakemore, Jr., et al., "Semiconductor Reliability - Final Report," ARINC Research Publication No. 239-01-4-383, Contract N0bsr-87664, ARINC Research Corp., 31 July 1963.

G. Chernowitz, et al., "Electromechanical Component Reliability," RADC Document No. TDR-63-295, DDC No. AD 422327, RADC Contract No. AF 30(602)-2652, American Power Jet Co., May 1963.

G. Chernowitz, et al., "Reliability Prediction for Mechanical and Electromechanical Parts," RADC Document No. TDR-64-50, RADC Contract No. AF 30(602)-2991, American Power Jet Co., January 1964.

H. R. Jeffers and H. S. Balaban, "Development of Prediction Techniques," System Reliability Prediction by Function, Vol. 1, ARINC Research Publication No. 241-01-1-375, RADC Document No. TDR-63-300, DDC No. AD 416494, RADC Contract No. AF 30(602)-2838, ARINC Research Corp., 27 May 1963.

H. G. Madderia, et al., "System Maintainability Measurement Study, Weapon System 107C, Titan II, Vol. IV," Martin Document No. CR 63-155, Contract No. AF 04(647)-576, Martin Co., August 1963.

R. Tommaney, "System Reliability Prediction by Function," RADC Document No. FADC TDR-63-146, DDC No. AD 406191, RADC Contract No. AF 30(602)-2687, Federal Electric Corp., May 1963.

Aerospace Corp., "Survey of Component Part Failure Rates," Aerospace Report 1923-1-89.

"Aerospace Maintenance and Operational Status (AMOS) Summary," an AFLC reporting system requiring monthly reports from AFLC Air Materials Areas (AMA's) which includes mean-time-between-maintenance (MTBM) and time-to-repair information obtained from AFM 66-1 data.

ARINC Research Corp., "Fundamentals of Reliability Prediction," Reliability Theory and Practice, Chapter 9, ARINC Research Publication No. 191-1-335, 1962.

ASTRO Reliability Corp., Astro Reliability Handbook, RD 101, AF Prime Contract AF 04(694)-1, Subcontract No. 56-204-SC, 17 July 1961.

ASTRO Reliability Corp., Reliability Assessment Handbook, 5 January 1963.

Autonetics, Div. of North American Aviation, Minuteman Standard Parts Handbook (WS-133A and B), Revision C, Publication No. 550-X-38, Contract AF 04(647)-923 for WS-133A and Contract AF 04(694)-247 for WS-133B, 5 April 1963.

Avco Corp., "Reliability Physics," March 1962; "Failure Rate," April 1962; "Failure Criteria," March 1962; "Failure Mechanisms," April 1962; "Reliability in Space Environment," Reliability Engineering Data Series, Avco Corp., Wilmington, Mass., January 1963.

Bureau of Ships Reliability Design Handbook, NavShips 94501, 29 March 1963.

EIA Engineering Department, (M-5.2 Publication), "Component Parts Failure Data Compendium," Reliability Bulletin No. 3, December 1962.

Lockheed Georgia Co., Reliability Reports, C-130A (period of January 1961 - December 1962); C-130B (Report period of January 1961 - August 1963); C-130E (Report period of April 1962 - July 1963, Operations Reliability and Maintainability Summary, Marietta, Georgia.

Martin-Marietta, "Reliability Analysis Guide," Martin Report No. ER-12208, December 1961.

Military Standardization Handbook No. 217, Reliability Stress and Failure Rate Data for Electronic Equipment.

RADC Reliability Notebook, RADC No. TR-58-111, OTS Nos. PB-161894, Basic, October 1959; PB-161894-2, Rev. 2, December 1961; PB-161894-3, Rev. 3, January 1963.

RCA Service Co., "Maintainability Engineering," Phase V, Vol. II, RADC Document No. RADC-TDR-63-85, DDC No. AD 404898, RADC Contract No. AF 30(602)-2057, 5 February 1963.

RCA Service Co., "Maintainability Measurement and Prediction Methods for AF Ground Electronic Equipment," Field Data Collection, Phase II, DDC No. AD 247155, Contract No. AF 30(602)-2057, 15 September 1960.

RCA Service Co., "Maintainability Technique Study," Final Tech. Report, Phase V, Vol. I, RADC Document No. RADC-TDR-63-85, DDC No. AD 404899, Contract No. AF 30(602)-2057, 5 February 1963.

U.S. Naval Ordnance Lab., Bureau of Naval Weapons Failure Rate Data Handbook, Corona, California, FARADA Handbook SP-63-470, Vol. 3, 1 June 1962, (Revised through 1 March 1964).

Vitro Corp., Handbook for the Prediction of Shipboard and Shore Electronic Equipment Reliability, Vitro Tech. Rpt No. 133, NavShips Contract 93820, April 1961.

Vitro Lab., "Reliability Analysis and Prediction Techniques," Vitro Tech.  
Note 1582.01-2, 11 February 1963.

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